Online Appendix for "Communication and Decision-Making in Corporate Boards"

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This online Appendix presents several extensions of the basic model. Section I considers two extensions that capture directors' desire to support the CEO. Section II analyzes a general linear specification of the decision-making rule. Finally, Section III considers a model with voting. All proofs are collected in Section IV.

I. Conformity due to fear of the CEO

Fear of CEO retaliation is one of the important reasons for pressure for conformity in the boardroom. In this section, I analyze two extensions that incorporate fear of the CEO into directors' preferences. I show that in both extensions, similarly to the basic model, pressure for conformity at the decision-making stage can have an overall positive effect on firm value because it encourages more communication.

I.A. Asymmetric conformity biases

In this extension, I assume that it is particularly costly for a director to deviate from other board members when he criticizes the CEO than when he supports the CEO. I show that the results of the basic model continue to hold in this setting as well.

Specifically, suppose that if a is the ultimate action chosen by the board, the CEO favors a higher action over a lower action. For example, in the context of investment decisions, the CEO could have a bias towards more investment (higher a) due to empire-building preferences. Suppose that director i's utility, given the final action a and other directors' actions a_j, $j \neq i$, is given by

$$U_i\left(a, a_1, ..., a_N, \theta\right) = \begin{cases} -\left(a-\theta\right)^2 - r_i\left(\bar{a}_{-i} - a_i\right) & , \textit{if } \bar{a}_{-i} > a_i \\ -\left(a-\theta\right)^2 & , \textit{if } \bar{a}_{-i} < a_i. \end{cases}$$

In other words, the director suffers a loss if he is less supportive of the manager than the average director ($a_i < \bar{a}_{-i}$) but does not suffer any loss if he is more supportive of the manager than others. Other assumptions of the basic model remain unchanged. Because this specification of directors' preferences makes the model less tractable, I focus on the case of two symmetric directors ($c_i = c$, $r_i = r$) and a uniform distribution of signals.

As in the basic model, directors' communication strategy takes a threshold form: a signal is revealed if and only if it lies outside the interval $[t, T]$ for some t, T. The next result derives the equilibrium at the decision-making stage taking this communication strategy as given.

Lemma B.1. *Suppose that at the communication stage, x_i is revealed if and only if $x_i \notin [t, T]$. Then the following strategies constitute an equilibrium at the decision-making stage.*

1. *If both signals were communicated, then $a_1^* = a_2^* = x_1 + x_2$.*
2. *If no signal was communicated, then*

$$a_i^* = \left(x_i + \frac{t+T}{2}\right) + \frac{r}{T-t}\left(T - x_i\right). \tag{B.1}$$

3. *If x_1 was communicated and x_2 was not, then $a_1^* = x_1 + A$, and*

$$a_2^* = \begin{cases} x_1 + x_2 & , \text{ if } x_2 > A \\ x_1 + A & , \text{ if } x_2 \in [A - r, A] \\ x_1 + x_2 + r & , \text{ if } x_2 < A - r, \end{cases} \tag{B.2}$$

where $A = \frac{\frac{t+T}{2} + \frac{rT}{T-t}}{1 + \frac{r}{T-t}} \in \left(\frac{t+T}{2}, T\right)$.

The intuition is the following. If both signals were revealed, directors coordinate on the optimal action $x_1 + x_2$, which maximizes firm value. Even if $x_1 + x_2$ is low and hence the board's decision goes against the CEO's preferences, directors do not suffer any loss because they jointly oppose the CEO. However, if a director did not reveal his signal, the other director does not know his view on the optimal decision and is afraid to be the one who is less supportive of the CEO. Fear of the CEO then induces the other director to bias his action upwards. In particular, if director 2 did not reveal his signal, then instead of the action $\mathbb{E}[\theta|I_1] = x_1 + \frac{t+T}{2}$, which maximizes firm value given his information, director 1's action is either $(x_1 + \frac{t+T}{2}) + \frac{r}{T-t}(T - x_1)$ or $x_1 + A$, which are both greater than $x_1 + \frac{t+T}{2}$.

Next, consider directors' communication strategy. Because a director is punished for being less supportive of the manager than the other director, a director with a negative signal has particularly strong incentives to share it with the other director prior to voting, when the manager is not present. By privately sharing his negative signal with the other director, he can avoid the cost of retaliation because he ensures that the other director will also oppose the manager at the decision-making stage. The following lemma confirms this intuition.

Lemma B.2. *A threshold equilibrium at the communication stage takes the following form: director i communicates his signal if and only if* $x_i \leq t$ for some $t \in [-k, k]$.

As in the basic model, a stronger conformity bias r has two opposite effects on firm value. First, directors distort their actions more if they did not share information with each other. Second, they are also more likely to share information. The intuition behind the positive effect of conformity on communication is similar to the basic model. When pressure for conformity is strong, a director with negative information about the CEO's preferred decision will not vote against it unless he is confident that other directors share his concerns. Thus, to be able to vote against the CEO without suffering retaliation, the director has incentives to convince others of his position by sharing his information before the vote. Because directors share their negative information beforehand, they can be more effective in jointly opposing the CEO than if each of them voted individually based on his private information.

To see this formally, note that (B.1) and (B.2) imply the following properties, which are similar to the basic model: first, if a signal was communicated, it is used efficiently by both directors, and second, if a signal was not communicated, conformity induces the director to use it inefficiently. Stronger pressure for conformity increases inefficiency from not communicating the signal and thus gives directors stronger incentives to incur the costs of communication. The next result shows that this positive effect dominates when r is sufficiently small.

Proposition B.1. *Firm value is maximized at a strictly positive value of r.*

Thus, similarly to the basic model, some degree of conformity is beneficial for firm value due to its positive effect on pre-vote communication.

I.B. Conformity to the CEO

In this extension, I assume that the CEO is one of the directors and focus on directors' desire to conform to the CEO. Consider the following variation of the setup. Let the CEO correspond to director 1 in the model. Denote the CEO's bias by b and assume, without loss of generality, that $b > 0$. For example, if a is the amount of investment in a project, a positive bias corresponds to empire-building preferences. Suppose that other directors are identical and have no directional biases: $b_i = 0$, $r_i = r$, $c_i = c > 0$, $k_i = k$ for $i > 1$. To emphasize the importance of conforming to the CEO's position, consider the following preferences:

$$\begin{aligned} U_1(a, \theta) &= -(a - (b + \theta))^2, \\ U_i(a, a_1, a_i, \theta) &= -(a - \theta)^2 - r(a_i - a_1)^2, i > 1. \end{aligned}$$

This specification implies that the CEO does not care about conforming to others ($r_1 = 0$), while directors only care about conforming to the CEO. Suppose also that the CEO's costs of communication are zero. Finally, in the notations of Section 3, suppose that the board's decision equals a_i with probability p_i, where p_1 is the CEO's decision-making power and $p_i = p = \frac{1-p_1}{N-1}$ is the decision-making power of each of the other directors.

Repeating the arguments of Lemma A.1 in the Appendix, it can be shown that the equilibrium strategies at the decision-making stage are given by

$$a_i = g_i + a_i(p_1, ..., p_N),$$

where $g_1 = b$, $g_i = \frac{r}{p+r}b$ for $i > 1$, and $a_i(p_1, ..., p_N)$ is given by (10) for signals that were communicated and by (20) for signals that were not. Intuitively, the desire to conform to the CEO distorts directors' actions: they not only put less than optimal weight ($\frac{p}{p+r} < 1$) on their private signals, but also bias their decisions (by $\frac{r}{p+r}b$) towards more investment, i.e., a higher a. However, this distortion is smaller when the CEO is less influential (p_1 is lower).

The proof of Proposition B.2 shows that at the communication stage, directors reveal their signals if they are sufficiently low, $x_i < t$. Intuitively, since the CEO is biased towards over-investment, directors do not reveal information that further supports a high amount of investment. Importantly, t increases with r, and hence pressure for conformity at the decision-making stage improves communication. The following lemma shows that Proposition 3 continues to hold.

Proposition B.2. *Firm value is maximized at a strictly positive value of r.*

The implication of the lemma is that even when directors care exclusively about conforming to the CEO, open ballot voting (corresponding to a higher r) could be more efficient than secret ballot voting due to its positive effect on the pre-vote discussion. The intuition is similar to the basic model. Consider a director with negative private information about a project favored by the CEO. If voting is by secret ballot, the director will vote against the project knowing that his vote will not be observed. However, since the project is supported by the CEO, the director may be reluctant to voice his concerns prior to the vote, leading to inefficient communication. In contrast, if voting is by open ballot, the director will not vote against the project if he knows that the CEO still supports it. The director's concern about firm value will then induce him to express his reservations during the discussion and do his best to convince the CEO and other directors that the project should not be undertaken, even though voicing this opinion may be costly.

II. General linear model

This section provides the analysis of the model for the general linear specification of the function $h(a_1, ..., a_N)$, which aggregates individual directors' actions into the decision a taken by the board. Specifically, the general linear specification is characterized by m linear combinations $(\gamma_{1(j)}, ...\gamma_{N(j)}), \gamma_{i(j)} \geq 0, \sum_{i=1}^{N} \gamma_{i(j)} = 1$, and probabilities $q_j > 0, \sum_{j=1}^{m} q_j = 1$, attached to these combinations, such that $h(a_1, ...a_N)$ is equal to $\sum_{i=1}^{N} \gamma_{i(j)} a_i$ with probability q_j. Then firm value is given by

$$V_0 - \sum_{j=1}^{m} q_j \left(\sum_{k=1}^{N} \gamma_{k(j)} a_k - \theta \right)^2 .$$

Suppose that the utility of director i is

$$U_i(a, \theta) = -\sum_{j=1}^{m} q_j \left(\sum_{k=1}^{N} \gamma_{k(j)} a_k - b_i - \theta \right)^2 - r_i \sum_{k \neq i} w_{k(i)} (a_i - a_k)^2 - \varphi_i (a_i - \theta)^2 ,$$

where $\sum_{k \neq i} w_{k(i)} = 1$. The first term reflects the directional bias b_i, the second term reflects the conformity bias, and the third term reflects the effect of the director's actions on his reputation - the director benefits from his own action being close to the shareholders' optimal action, even if the action taken by the board is sufficiently different from this optimal action.

Denote by I_i the director's information set after the communication stage. Taking the first-order condition,

$$a_i = \frac{\sum_j q_j \gamma_{i(j)}}{\varphi_i + r_i + \sum_j q_j \gamma_{i(j)}^2} b_i + \frac{\varphi_i + \sum_j q_j \gamma_{i(j)}}{\varphi_i + r_i + \sum_j q_j \gamma_{i(j)}^2} \mathbb{E}\left[\theta | I_i\right] + \sum_{k \neq i} \frac{r_i w_{k(i)} - \sum_j q_j \gamma_{i(j)} \gamma_{k(j)}}{\varphi_i + r_i + \sum_j q_j \gamma_{i(j)}^2} \mathbb{E}\left[a_k | I_i\right].$$

Denote

$$\begin{aligned} F_i &= \frac{\sum_j q_j \gamma_{i(j)}}{\varphi_i + r_i + \sum_j q_j \gamma_{i(j)}^2}, \\ Q_i &= \frac{\varphi_i + \sum_j q_j \gamma_{i(j)}}{\varphi_i + r_i + \sum_j q_j \gamma_{i(j)}^2}, \\ Z_{k(i)} &= \frac{r_i w_{k(i)} - \sum_j q_j \gamma_{i(j)} \gamma_{k(j)}}{\varphi_i + r_i + \sum_j q_j \gamma_{i(j)}^2}. \end{aligned} \tag{B1}$$

Using the condition $\sum_{i=1}^{N} \gamma_{i(j)} = 1$, it is straightforward to check that

$$Q_i + \sum_{k \neq i} Z_{k(i)} = 1. \tag{B2}$$

Then, the first-order condition can be written as

$$a_i = F_i b_i + (1 - \sum_{k \neq i} Z_{k(i)}) \mathbb{E}\left[\theta | I_i\right] + \sum_{k \neq i} Z_{k(i)} \mathbb{E}\left[a_k | I_i\right].$$

The following results repeat the results of the basic model for this general specification.

Lemma C.1 (equilibrium at the decision-making stage). *Suppose that at the communication stage signals $x_i, i \in J_C$ were communicated and that y_i is the expected value of x_i conditional on no communication. Denote $J_{NC} = \{1, ..., N\} \backslash J_C$, $X = \sum_{i \in J_C} x_i$, and $Y = \sum_{i \in J_{NC}} y_i$. Then there is a linear equilibrium at the decision-making stage characterized by the following strategies:*

1. If director i communicated his signal, $i \in J_C$, his action is given by

$$a_i = g_i + X + Y.$$

2. If director i did not communicate his signal, $i \in J_{NC}$, his action is given by

$$a_i = g_i + X + Y + Q_i \left(x_i - y_i\right),$$

where g_i solves the linear system

$$g_i = F_i b_i + \sum_{k \neq i} Z_{k(i)} g_k, \tag{B3}$$

and $F_i, Q_i, Z_{k(i)}$ are given by (B1) and satisfy (B2).

Lemma C.2 (equilibrium at the communication stage). *Suppose that conditional on director i not communicating his signal, other directors believe that the expected value of x_i is y_i. Then director i has incentives to communicate x_i if and only if it satisfies $H_i(x_i - y_i) > 0$, where*

$$H_i(\delta) = A_i \delta^2 - 2B_i \delta - c_i,$$

where

$$\begin{aligned} A_i &= \sum_{j=1}^{m} q_j \left[1 - \gamma_{i(j)} Q_i\right]^2 + r_i Q_i^2 + \varphi_i (1 - Q_i)^2, \\ B_i &= \sum_{j=1}^{m} q_j \left[1 - \gamma_{i(j)} Q_i\right] \left[\sum_{k=1}^{N} \gamma_{k(j)} g_k - b_i\right] - r_i Q_i \left(g_i - \sum_{n \neq i} w_{n(i)} g_n\right) + \varphi_i g_i (1 - Q_i), \end{aligned}$$

Q_i is given by (B1), and g_i solves (B3).

Lemma C.3 (expected firm value). *Suppose that at the communication stage director i communicates his signal if and only if $x_i \in C_i$, and let $y_i = \mathbb{E}[x_i | x_i \notin C_i]$. Then expected firm value is given by*

$$V_0 - \mathbb{E}\varepsilon^2 - \sum_{j=1}^{m} q_j \left(\sum_{i=1}^{N} \gamma_{i(j)} g_i\right)^2 - \sum_{k=1}^{N} \left[\sum_{j=1}^{m} q_j \left(\gamma_{k(j)} Q_k - 1\right)^2\right] \int_{x_k \notin C_k} (x_k - y_k)^2 f_k(x_k)\, dx_k,$$

where Q_i is given by (B1) and g_i solves (B3).

Lemma C.4 (inefficient decision rules) *Suppose that $h(a_1, ..., a_N)$ is non-deterministic, that is, there exists i such that $\gamma_{i(j)} \neq \gamma_{i(j')}$ for some j, j', where $q_j > 0$, $q_{j'} > 0$. Then, $h(a_1, ..., a_N)$ does not efficiently aggregate information according to Definition 1.*

Proposition C.1. *Suppose that $h(a_1, ..., a_N)$ is non-deterministic, that is, there exists i such that $\gamma_{i(j)} \neq \gamma_{i(j')}$ for some j, j', where $q_j > 0$, $q_{j'} > 0$. Suppose also that $b_k = \varphi_k = 0$*

for all k, and $r_k = 0$ for $k \neq i$. Then firm value is maximized at $r_i^* > 0$..

III. Model with voting

This section provides the analysis of a model with voting. The board, which consists of two directors, is contemplating a proposal. The value of the firm is equal to

$$V(a,\theta) = -(a-\theta)^2,$$

where $a \in \{1,-1\}$ is the decision made by the board, and $\theta = x_1 + x_2$ is the unknown state of the world. Decision $a = 1$ ($a = -1$) corresponds to the board accepting (rejecting) the proposal. At the decision-making stage, each director casts a vote $a_i \in \{1,-1\}$, where $a_i = 1$ ($a_i = -1$) corresponds to voting in favor of (against) the proposal. The decision-making rule is as follows. If both directors vote in favor of (against) the proposal, it is accepted (rejected) with probability 1. If directors disagree with each other, the proposal is accepted with probability 0.5.

Signals x_1, x_2 are independent and are uniformly distributed on $[-1,1]$. Director i perfectly observes x_i but has no information about the other signal. The timeline is as follows. At the communication stage, directors simultaneously decide whether to reveal their signals to each other. Communicating the signal entails a cost $c > 0$. Then, at the decision-making stage, directors simultaneously cast their votes, and the board's decision is determined.

Each director cares about shareholder value. In addition, directors have a conformity bias: they incur a loss if their vote deviates from the vote of the other director. In particular, a director's utility function is given by

$$U_i(a_i, a_j, a, \theta) = V(a,\theta) - r \cdot 1\{a_i \neq a_j\}.$$

Given the symmetry of the setup, we conjecture that the equilibrium at the communication stage takes the following threshold form: there exists a threshold d such that signal x_i is revealed if and only if $|x_i| > d$. We later verify this conjecture.

First, consider the equilibrium at the decision-making stage. There are three possible scenarios depending on the outcome of the communication stage: 1) no signal was revealed; 2) only one signal was revealed; 3) both signals were revealed. The following proposition characterizes the equilibrium.

Proposition D1.

1. *If no signal was revealed at the communication stage, director i votes for the proposal if and only if $x_i > 0$.*

2. *If only signal x_i was revealed at the communication stage, director i votes for the proposal if and only if $x_i > 0$. If $x_i > 0$, director j votes for the proposal if and only if $x_j > -\frac{r}{2} - x_i$. If $x_i < 0$, director j votes for the proposal if and only if $x_j > \frac{r}{2} - x_i$.*

3. *If both signals were revealed at the communication stage, both directors vote for the proposal if and only if $x_1 + x_2 > 0$.*

Given this equilibrium at the decision-making stage, we next verify the conjectured equilibrium at the communication stage and find the communication threshold d.

Proposition D2. *At the communication stage, director i reveals his signal x_i if and only if $|x_i| > d$, where $d = \frac{-r+\sqrt{r^2+8c}}{2}$.*

Since $\frac{-r+\sqrt{r^2+8c}}{2}$ decreases in r, the result of the basic model continues to hold: stronger conformity biases improve communication between directors.

IV. Proofs

Proofs of Section I

Proof of Lemma B.1. Given the "random dictator" rule, the director's utility can be rewritten as

$$U_i(a, a_1, ..., a_N, \theta) = \begin{cases} -\frac{1}{N}\sum_{j=1}^{N}(a_j - \theta)^2 - r_i(\bar{a}_{-i} - a_i) & , \textit{if } \bar{a}_{-i} > a_i \\ -\frac{1}{N}\sum_{j=1}^{N}(a_j - \theta)^2 & , \textit{if } \bar{a}_{-i} < a_i. \end{cases}$$

Consider each of the following three cases separately.

(1) Suppose that both signals were communicated.

The actions $a_1^* = a_2^* = x_1 + x_2$ constitute an equilibrium because the utility of both directors is equal to zero, which is the global maximum, and hence no profitable deviation exists. There also exist other equilibria. In unreported results, I prove that all possible equilibria take the form $a_1^* = a_2^* = a^*$ for some $a^* \in [x_1 + x_2, x_1 + x_2 + r]$. I select the most efficient of these equilibria, $a^* = x_1 + x_2$, which maximizes firm value, but the results are robust to the equilibrium selection.

(2) Suppose that signals $x_1, x_2 \in [t, T]$ and hence were not communicated.

Denote $\gamma = 1 - \frac{r}{T-t}$ and $\beta = \frac{t+T}{2} + \frac{Tr}{T-t}$. We next prove that the best response of director 1 to the strategy $a_2 = \gamma x_2 + \beta$ of director 2 indeed takes the form $a_1 = \gamma x_1 + \beta$ if $x_1 \in [t, T]$. Note that $a_2 > a_1 \Leftrightarrow x_2 > \frac{1}{\gamma}(a_1 - \beta)$. Consider the following three regions for a_1:

- First, if $a_1 > \gamma T + \beta = T + \frac{t+T}{2}$, then $a_2 < a_1$ for all $x_2 \in [t, T]$, and hence (up to a constant), $U_1 = -\frac{1}{2}(a_1 - \theta)^2$, which is an inverted parabola that has a maximum at $\mathbb{E}[\theta] = x_1 + \frac{T+t}{2}$. Hence, the optimal action in this region is $\max(x_1 + \frac{T+t}{2}, T + \frac{t+T}{2})$.
- Second, if $a_1 < \gamma t + \beta = r + t + \frac{t+T}{2}$, then $a_2 > a_1$ for all $x_2 \in [t, T]$, and hence $\mathbb{E}[U_1]$ equals

$$-\frac{1}{2}\mathbb{E}(a_1 - \theta)^2 - \frac{r}{T-t}\int_t^T (\gamma x_2 + \beta - a_1)\, dx_2 = -\frac{1}{2}\mathbb{E}(a_1 - \theta)^2 + r\left(a_1 - \gamma\frac{T+t}{2} - \beta\right).$$

  Note that $\mathbb{E}[U_1]' > 0 \Leftrightarrow a_1 < r + x_1 + \frac{T+t}{2}$, and hence the optimal action in this region is $\min\left(r + x_1 + \frac{t+T}{2}, r + t + \frac{t+T}{2}\right)$.
- Third, if $\gamma t + \beta < a_1 < \gamma T + \beta$, then $\mathbb{E}[U_1]$ equals

$$-\frac{1}{2}\mathbb{E}(a_1 - \theta)^2 - \frac{r}{T-t}\int_{\frac{1}{\gamma}(a_1-\beta)}^T (\gamma x_2 + \beta - a_1)\, dx_2 = -\frac{\mathbb{E}(a_1 - \theta)^2}{2} - \frac{r}{\gamma}\frac{(\gamma T + \beta - a_1)^2}{2(T-t)},$$

  and $\mathbb{E}[U_1]' > 0 \Leftrightarrow a_1\left(1 + \frac{r}{T-t}\frac{1}{\gamma}\right) < x_1 + \frac{T+t}{2} + \frac{r}{T-t}\frac{1}{\gamma}(\gamma T + \beta) \Leftrightarrow a_1 < \gamma x_1 + \beta$. Hence, the optimal action in this region is $[\gamma x_1 + \beta]_{\gamma t+\beta}^{\gamma T+\beta}$, where $[x]_a^b$ is x truncated by a from below and by b from above.

Summarizing these three cases, it can be shown that the best response of director 1 is $a_1 = [\gamma x_1 + \beta]_{\gamma t+\beta}^{\gamma T+\beta}$, which coincides with the conjectured strategy $\gamma x_1 + \beta$ when $x_1 \in [t, T]$.

(3) Suppose that x_1 was communicated and x_2 was not. First, let us find the best response of director 2 to $a_1 = x_1 + A$. Consider the following two regions for a_2:

- If $a_2 > x_1 + A$, $\mathbb{E}[U_2] = -\frac{1}{2}\mathbb{E}(a_2 - \theta)^2$, which is an inverted parabola that has a maximum at $a_2 = x_1 + x_2$. Hence, the optimal action in this region is $\max(x_1 + x_2, x_1 + A)$.
- If $a_2 < x_1 + A$, $\mathbb{E}[U_2] = -\frac{1}{2}\mathbb{E}(a_2 - \theta)^2 - r(a_1 - a_2)$, which is an inverted parabola with a maximum at $a_2 = x_1 + x_2 + r$. Hence, the optimal action in this region is $\min(x_1 + x_2 + r, x_1 + A)$.

  Summarizing the two cases, it can be shown that the best response of director 2 is (B.2).

Second, let us find the best response of director 1 to a_2 given by (B.2). Consider the following two regions for a_1:

- If $a_1 > x_1 + A$, then $a_2 > a_1$ if and only if (1) $x_2 > A$ and (2) $x_2 > a_1 - x_1\,(> A)$. Any $a_1 > x_1 + T$ is dominated by $a_1 = x_1 + T$ because $a_2 > a_1$ in both cases, but $x_1 + T$ is closer to the optimal action $\mathbb{E}[\theta]$. For $a_1 \leq x_1 + T$,

$$\mathbb{E}[U_1] = -\frac{1}{2}\mathbb{E}(a_1 - \theta)^2 - \frac{1}{2}\frac{r}{T-t}(T + x_1 - a_1)^2,$$

  which is an inverted parabola that has a maximum at $a_1 = x_1 + \frac{\frac{T+t}{2} + \frac{rT}{T-t}}{1 + \frac{r}{T-t}} = x_1 + A$. Hence, the optimal action in this region is $x_1 + A$.

- If $a_1 < x_1 + A$, then

$$\mathbb{E}[U_1] = -\frac{1}{2}\mathbb{E}(a_1 - \theta)^2 - \frac{r}{T-t}[\frac{(T + x_1 - a_1)^2}{2} + (x_1 + A - a_1)\,r],$$

  which is an inverted parabola that has a maximum at $a_1 = x_1 + A + \frac{r^2}{T-t+r} > x_1 + A$. Hence, the optimal action in this region is also $x_1 + A$.

  Summarizing these two cases, the best response of director 1 is indeed $x_1 + A$.

Proof of Lemma B.2. Let $[t, T]$ be the equilibrium non-communication region. Consider the best response of director 1 to this communication strategy of director 2. Let $U_1^C(x_1, x_2)$, $U_1^{NC}(x_1, x_2)$ be the utility of director 1 when he communicates and does not communicate his signal, respectively, given signal x_2 of the other director. Also let $U_1^C(x_1) = \mathbb{E}_{x_2}\left[U_1^C(x_1, x_2)\right]$ and $U_1^{NC}(x_1) = \mathbb{E}_{x_2}\left[U_1^{NC}(x_1, x_2)\right]$ be the expected utility of director 1 from communicating and not communicating his signal, where the expectation is taken over all possible realizations of x_2. Then director 1 chooses to communicate his signal if and only if $\Delta(x_1) > 0$, where

$$\Delta(x_1) = U_1^C(x_1) - U_1^{NC}(x_1) - c.$$

By continuity of the best response and utility functions, if t (or T) are interior points, it must be that $\Delta(t) = 0$ ($\Delta(T) = 0$). Below I prove that

$$\Delta(t) > \Delta(T). \tag{1}$$

The statement of the lemma follows from this inequality. Indeed, t and T cannot both be interior points because otherwise, $\Delta(t) = \Delta(T) = 0$, contradicting (1). Similarly, if $t = -k$ and $T \in (-k, k)$, then $\Delta(T) = 0$ and hence $\Delta(t) > 0$, implying that the director prefers to communicate his signal at $x_1 = -k$. Thus, the only possible case is $T = k$ and $t \in [-k, k]$. If c is very large, directors never communicate their signals ($t = -k$), and when c converges to zero, there is full communication in the limit ($t \to k$).

Proof that $\Delta(t) > \Delta(T)$.

First, note that $U_1^C(x_1)$ does not depend on x_1. Indeed, by Lemma B.1, $U_1^C(x_1, x_2) = 0$ if $x_2 \notin [t, T]$. If $x_2 \in [t, T]$, x_2 is not revealed and hence $a_1 = x_1 + A$, a_2 is given by (B.2), and since $\theta = x_1 + x_2$, then $(a_1 - \theta)^2$, $(a_2 - \theta)^2$, and $(a_2 - a_1)^+$ do not depend on x_1. Thus, for any $x_2, U_1^C(x_1, x_2)$ does not depend on x_1, and hence $U_1^C(x_1)$ does not depend on x_1 either.

Thus, it remains to prove that $U_1^{NC}(t) < U_1^{NC}(T)$. Possible values of x_2 fall into two regions: $x_2 \notin [t, T]$ and $x_2 \in [t, T]$.

1) First, if $x_2 \notin [t, T]$, director 2 communicates his signal. Then $a_2 = x_2 + A$, and according to the proof of Lemma B.1, the optimal action of 1 is given by (B.2). Hence,

$$U_1^{NC}(x_1, x_2) = \begin{cases} -\frac{1}{2}(A - x_1)^2 & , \textit{if } x_1 > A \\ -\frac{1}{2}(A - x_1)^2 - \frac{1}{2}(A - x_1)^2 & , \textit{if } x_1 \in [A - r, A] \\ -\frac{1}{2}(A - x_1)^2 - \frac{1}{2}r^2 - r(A - r - x_1) & , \textit{if } x_1 < A - r. \end{cases}$$

This function increases below A, reaches a maximum of zero at $x_1 = A$, and then decreases. Recall that $A \in \left(\frac{t+T}{2}, T\right)$ and that t, T are equally distanced from $\frac{T+t}{2}$. Hence, $t < A < T$ and $T - A < A - t$. Therefore, $t < 2A - T$, and hence $U_1^{NC}(t, x_2) < U_1^{NC}(2A - T, x_2)$. In turn, $U_1^{NC}(2A - T, x_2) < U_1^{NC}(T, x_2)$ because the term $-\frac{1}{2}(A - x_1)^2$ takes the same value in the two points, and the terms $-\frac{1}{2}(A - x_1)^2$ and $-\frac{1}{2}r^2 - r(A - r - x_1)$ are strictly negative. Therefore, $U_1^{NC}(t, x_2) < U_1^{NC}(2A - T, x_2) < U_1^{NC}(T, x_2)$.

2) Second, if $x_2 \in [t, T]$, director 2 does not communicate his signal, and $a_2 = \left(x_2 + \frac{t+T}{2}\right) + \frac{r}{T-t}(T - x_2) = \gamma x_2 + \beta$, where $\gamma = 1 - \frac{r}{T-t}$ and $\beta = \frac{t+T}{2} + \frac{Tr}{T-t}$. According to the proof of Lemma B.1, $a_1(T) = \gamma T + \beta \geq a_2$ and $a_1(t) = \gamma t + \beta \leq a_2$. Hence:

$$\begin{aligned} U_1^{NC}(T, x_2) &= -\frac{1}{2}(T + x_2 - \gamma x_2 - \beta)^2 - \frac{1}{2}(T + x_2 - \gamma T - \beta)^2, \\ U_1^{NC}(t, x_2) &= -\frac{1}{2}(t + x_2 - \gamma x_2 - \beta)^2 - \frac{1}{2}(t + x_2 - \gamma t - \beta)^2 - r\gamma(x_2 - t). \end{aligned}$$

Then

$$\begin{aligned} H_1(T) \equiv -2\int_t^T U_1^{NC}(T, x_2)\,dx_2 &= \int_t^T [(T - \beta + x_2(1-\gamma))^2 + (T(1-\gamma) + x_2 - \beta)^2]dx_2 \\ &= \frac{1}{3}\frac{(T-\beta+T(1-\gamma))^3 - (T-\beta+t(1-\gamma))^3}{(1-\gamma)} + \frac{1}{3}\frac{(T-\beta+T(1-\gamma))^3 - (t-\beta+T(1-\gamma))^3}{1} \end{aligned}$$

and

$$\begin{aligned} H_1(t) \equiv -2\int_t^T U_1^{NC}(t, x_2)\,dx_2 &= \int_t^T [(t - \beta + x_2(1-\gamma))^2 + (t(1-\gamma) + x_2 - \beta)^2 \\ +2r\gamma(x_2 - t)]dx_2 &= \frac{1}{3}\frac{(t-\beta+T(1-\gamma))^3 - (t-\beta+t(1-\gamma))^3}{(1-\gamma)} + \frac{1}{3}\frac{(T-\beta+t(1-\gamma))^3 - (t-\beta+t(1-\gamma))^3}{1} + \frac{2r\gamma(T-t)^2}{2}. \end{aligned}$$

Using the expressions for γ and β and denoting $\frac{T-t}{2} = d$, we get $T - \beta + T(1-\gamma) = d$, $T - \beta + t(1-\gamma) = d - r$, $t - \beta + T(1-\gamma) = -d$, and $t - \beta + t(1-\gamma) = -d - r$. Hence,

$$\begin{aligned} 3H_1(T) - 3H_1(t) &= \frac{d^3 - (d-r)^3 - (-d)^3 + (-d-r)^3}{(1-\gamma)} + \frac{d^3 - (-d)^3 - (d-r)^3 + (-d-r)^3}{1} - 3r\gamma(T-t)^2 \\ &= -6dr^2\left(\frac{1}{1-\gamma} + 1\right) - 3r\gamma(T-t)^2 < 0, \end{aligned}$$

and hence, $\int_t^T U_1^{NC}(T, x_2)\,dx_2 > \int_t^T U_1^{NC}(t, x_2)\,dx_2$. Combining the results for the two ranges of x_2, we get the inequality $U_1^{NC}(t) < U_1^{NC}(T)$.

Proof of Proposition B.1. We show that the derivative of firm value at $r = 0$ is strictly positive, i.e., the positive effect of r on communication exceeds its negative effect at the decision-making stage. Using the notations in the proof of Lemma B.2, let $d \equiv \frac{T-t}{2}$ and recall from Lemma B.2 that $T = k$. Consider firm value $V(r, d)$ as a function of r and d. It satisfies:

$$-2V(r,d) = \mathbb{E}_{x_1,x_2}\left[(a_1 - x_1 - x_2)^2 + (a_2 - x_1 - x_2)^2\right].$$

Using Lemma B.1 and B.2, the following three cases are relevant:

1. If $x_i \notin [t, T]$ for $i = 1, 2$, both signals are revealed and $a_i = x_1 + x_2$.

2. If $x_i \in [t, T]$ for $i = 1, 2$, neither signal is revealed, and $a_i = \gamma x_i + \beta$, so $(a_i - x_1 - x_2)^2 = ((1-\gamma)x_i + x_j - \beta)^2$

3. If $x_i \in [t, T]$ and $x_j \notin [t, T]$, $a_j = x_j + A$, and a_i is given by (B.2). For a very small r, $t < A - r$ because $A > \frac{T+t}{2}$. Hence,

$$-2V(r,d) = \frac{1}{4k^2}(2I_1 + 2I_2),$$

where

$$I_1 = \int_t^T \int_t^T ((1-\gamma)x_1 + x_2 - \beta)^2 \, dx_1 dx_2,$$
$$I_2 = \int_{x_1 \notin [t,T]} \left(\int_t^T (A - x_2)^2 dx_2 + \int_t^{A-r} (r)^2 dx_2 + \int_{A-r}^{A} (A - x_2)^2 dx_2 \right) dx_1.$$

Integrating over x_1, x_2, using $\gamma = 1 - \frac{r}{T-t} = 1 - \frac{r}{2d}$ and the properties derived in the proof of Lemma B.2, we get

$$\begin{aligned} I_1 &= \tfrac{[(1-\gamma)T+T-\beta]^4 - [(1-\gamma)T+t-\beta]^4 - [(1-\gamma)t+T-\beta]^4 + [(1-\gamma)t+t-\beta]^4}{3 \cdot 4 \cdot (1-\gamma)} \\ &= \tfrac{d}{r} \tfrac{[d]^4 - [-d]^4 - [d-r]^4 + [d+r]^4}{6} = \tfrac{4}{3}\left(d^4 + d^2 r^2\right). \end{aligned}$$

Also, using the properties $T - A = \frac{d}{1+\frac{r}{2d}}$, $t - A = -\frac{d+r}{1+\frac{r}{2d}}$, and $A - r - t = \frac{d - \frac{r^2}{2d}}{1+\frac{r}{2d}}$, we get

$$I_2 = 2(k-d)\left(\frac{d^3 + (d+r)^3}{3\left(1+\frac{r}{2d}\right)^3} + r^2 \frac{d - \frac{r^2}{2d}}{1+\frac{r}{2d}} + \frac{r^3}{3} \right).$$

Denote $[d'_r]_{r=0} = d'$, which, as shown below, is strictly negative and finite. Then,

$$\left[\tfrac{d}{dr} I_1\right]_{r=0} = \tfrac{16}{3} d^3 d',$$
$$\left[\tfrac{d}{dr} I_2\right]_{r=0} = \tfrac{2}{3}(k-d)\left[\tfrac{d}{dr} \tfrac{d^3 + (d+r)^3}{\left(1+\frac{r}{2d}\right)^3}\right]_{r=0} - \tfrac{4}{3} d^3 d' = \tfrac{2}{3}(k-d)\left[6d^2 d'\right] - \tfrac{4}{3} d^3 d'.$$

Combining together,

$$\frac{d}{dr}\left[-4k^2 V(r,d)\right]_{r=0} = 4d^3 d' + 4(k-d) d^2 d' = 4kd^2 d'.$$

Hence, to show that $\frac{d}{dr}[V(r,d)]_{r=0} > 0$, it is sufficient to show that $d' = [d'_r]_{r=0} < 0$.

Proof that $[d'_r]_{r=0} < 0$**.**

Equivalently, we want to show that $[t'_r]_{r=0} > 0$, where $[-k, t]$ is the communication region. Using the notations in the proof of Lemma B.1, t satisfies $U_1^C(t) - U_1^{NC}(t) - c = 0$. Consider the function $G(t, r) = U_1^C(t) - U_1^{NC}(t)$. By the implicit function theorem, the inequality $[t'_r]_{r=0} > 0$ follows from $\left[\frac{\partial G(t,r)}{\partial t}\frac{\partial G(t,r)}{\partial r}\right]_{r=0} < 0$. We will show that, indeed, $\left[\frac{\partial G(t,r)}{\partial t}\right]_{r=0} < 0$ and $\left[\frac{\partial G(t,r)}{\partial r}\right]_{r=0} > 0$. With a slight change of notations, let $d = \frac{k-t(d)}{2} \Leftrightarrow t(d) = k - 2d$ and $G(d, r) \equiv G(t(d), r)$. Then, we need to show $\left[\frac{\partial G(d,r)}{\partial d}\right]_{r=0} > 0$ and $\left[\frac{\partial G(d,r)}{\partial r}\right]_{r=0} > 0$.

If x_1 is revealed, then $a_1 = x_1 + A$, and using (B.2) and the derivations above,

$$\left[-2U_1^C(x_1)\right]_{x_1=t} \cdot 2k = \int_t^T (x_2 - A)^2 dx_2 + \int_t^{A-r} (r^2) dx_2 + \int_{A-r}^A (x_2 - A)^2 dx_2 + 2r\int_A^T (x_2 - A) dx_2$$
$$= \frac{(T-A)^3-(t-A)^3}{3} + r^2(A - r - t) + \frac{r^3}{3} + 2r\frac{(T-A)^2}{2} = \frac{d^3+(d+r)^3}{3\left(1+\frac{r}{2d}\right)^3} + r^2\frac{d-\frac{r^2}{2d}}{1+\frac{r}{2d}} + \frac{r^3}{3} + r\frac{d^2}{(1+\frac{r}{2d})^2},$$

and if $x_1 = t$ is not revealed, then

$$\left[-2U_1^{NC}(x_1)\right]_{x_1=t} \cdot 2k = \int_{x_2\notin[t,T]} \left((t - A)^2 + r^2 + 2r(A - t - r)\right) dx_2 +$$
$$+ \int_{x_2\in[t,T]} \left[((1 - \gamma)t + x_2 - \beta)^2 + ((1 - \gamma)x_2 + t - \beta)^2\right] dx_2 + 2r\int_t^T \gamma(x_2 - t) dx_2$$
$$= 2(k - d)\left[\frac{(d+r)^2}{(1+\frac{r}{2d})^2} + r^2 + 2r\frac{d-\frac{r^2}{2d}}{1+\frac{r}{2d}}\right] + \frac{8}{3}d^3 + 2d^2r + \frac{8}{3}dr^2 + r\left(1 - \frac{r}{2d}\right)4d^2.$$

Hence,

$$2G(d, r) \cdot 2k = 2(k - d)\left[\frac{(d+r)^2}{(1+\frac{r}{2d})^2} + r^2 + 2r\frac{d-\frac{r^2}{2d}}{1+\frac{r}{2d}}\right] + \frac{8}{3}d^3 + 2d^2r + \frac{8}{3}dr^2 + r\left(1 - \frac{r}{2d}\right)4d^2$$
$$-\frac{d^3+(d+r)^3}{3\left(1+\frac{r}{2d}\right)^3} - r^2\frac{d-\frac{r^2}{2d}}{1+\frac{r}{2d}} - \frac{r^3}{3} - r\frac{d^2}{(1+\frac{r}{2d})^2}.$$

Differentiating with respect to d, r, and setting $r = 0$:

$$4k \cdot \left[\frac{\partial G(d,r)}{\partial d}\right]_{r=0} = -2d^2 + 2(k - d)2d + 8d^2 - \frac{1}{3}6d^2 = 4d(k - d) + 4d^2 = 4dk > 0,$$
$$4k \cdot \left[\frac{\partial G(d,r)}{\partial r}\right]_{r=0} = 2(k - d)3d + 5d^2 - \frac{1}{3}\left[3(d + r)^2 - 2d^3\left(3\frac{1}{2d}\right)\right] = 6(k - d)d + 5d^2 > 0.$$

Hence, $[d'_r]_{r=0} < 0$, which completes the proof.

Proof of Proposition B.2. Consider the communication stage. Repeating the proof of Lemma A.3, it can be shown that director $i > 1$ has incentives to reveal x_i if and only if $H(x_i - y_i) > 0$, where

$$H(\delta) = \delta^2 - 2\delta b\left(\frac{pp_1 + r}{p(1 - p) + r}\right) - \frac{c}{1 - \frac{p^2}{p+r}}.$$

Since the coefficient for the linear term is negative, the proof of Lemma 3 implies that directors reveal their signals if and only if $x_i < k + 2\delta^-$, where $\delta^- < 0$ is the smallest root of $H(\cdot)$. Denoting $B = \frac{pp_1+r}{p(1-p)+r}b$ and $C = \frac{c}{1-\frac{p^2}{p+r}}$, we get $\delta^- = B - \sqrt{B^2+C}$ and $\frac{d\delta^-}{dr} = \frac{\partial\delta^-}{\partial B}B'_r + \frac{\partial\delta^-}{\partial C}C'_r$. Calculating these derivatives, it is straightforward to see that $B'_r > 0$, $C'_r < 0$, $\frac{\partial\delta^-}{\partial B} > 0$, $\frac{\partial\delta^-}{\partial C} < 0$, and hence $\frac{d\delta^-}{dr} > 0$. Thus, communication increases with r.

Using Lemma A.4 for a uniform distribution, expected firm value is

$$\mathbb{E}(V) = V_0 - \mathbb{E}\varepsilon^2 - \sum_i p_i g_i^2 - \sum_{i>1}\left[1 - p + p\left(\frac{r}{p+r}\right)^2\right]\frac{1}{3k}\left(\frac{T_i - t_i}{2}\right)^3,$$

where $[t_i, T_i] = \left[k + 2\delta^-, k\right]$ for $i > 1$. Hence, $\frac{T_i - t_i}{2} = -\delta^-$, and we can rewrite $\mathbb{E}(V)$ as

$$\mathbb{E}(V) = V_0 - \mathbb{E}\varepsilon^2 - b^2\left(p_1 + (1-p_1)\left(\frac{r}{p+r}\right)^2\right) + \frac{N-1}{3k}\left(1 - p + p\left(\frac{r}{p+r}\right)^2\right)\left(\delta^-\right)^3.$$

To prove the proposition, we show that $\lim_{r\to 0}\frac{d\mathbb{E}(V)}{dr} > 0$. Since $\lim_{r\to 0}\frac{d}{dr}\left(\frac{r}{p+r}\right)^2 = 0$, a sufficient condition is $\lim_{r\to 0}\frac{d\left(\delta^-\right)^3}{dr} > 0$. Note that $\lim_{r\to 0}B'_r > 0$, $\lim_{r\to 0}C'_r < 0$, $\lim_{r\to 0}\frac{\partial\delta^-}{\partial B} > 0$, $\lim_{r\to 0}\frac{\partial\delta^-}{\partial C} < 0$, and hence $\lim_{r\to 0}\frac{d\left(\delta^-\right)^3}{dr} = 3\lim_{r\to 0}\left(\delta^-\right)^2\left(\frac{\partial\delta^-}{\partial B}B'_r + \frac{\partial\delta^-}{\partial C}C'_r\right) > 0$.

Proofs of Section II

Proof of Lemma C.1. Plugging these strategies into the first-order condition derived above and using the fact that $\mathbb{E}[x_k - y_k|I_i]$ for $k \in J_{NC}, i \neq k$, we verify that these are indeed equilibrium strategies:

1) For $i \in J_C$:

$$a_i = F_i b_i + \left(1 - \sum_{k\neq i} Z_{k(i)}\right)(X+Y) + \sum_{k\neq i} Z_{k(i)}\left(g_i + X + Y\right) = g_i + X + Y.$$

2) For $i \in J_{NC}$:

$$a_i = F_i b_i + \left(1 - \sum_{k\neq i} Z_{k(i)}\right)(X + Y - y_i + x_i) + \sum_{k\neq i} Z_{k(i)}\left(g_i + X + Y\right) = g_i + X + Y + Q_i\left(x_i - y_i\right).$$

Proof of Lemma C.2. Suppose that the equilibrium communication and non-communication regions of director i are some sets C_i and NC_i, $C_i \cup NC_i = [-k_i, k_i]$. That is, the director communicates his signal x_i if and only if $x_i \in C_i$. Denote $y_i = \mathbb{E}[x_i|x_i \in NC_i]$ and $\delta_i = x_i - y_i$.

Consider the decision of director 1 with signal x_1 whether to pay c_1 to communicate his signal. The director does not know the signals of other directors and thus conditions his decision on all possible values of $x_2, ..., x_N$. Suppose that among the remaining signals, signals $x_i, i \in J_C$ lie in their respective regions C_j and are therefore communicated, and signals $x_i, i \in J_{NC}$ lie in their respective regions NC_j and are not communicated. It can be shown, using the equilibrium at the decision-making stage, that if the director communicates his signal, then his payoff upon communication, U_1^C, is equal to

$$\begin{gathered}-\sum_{j=1}^m q_j\left[\sum_{k\in J_{NC}}\delta_k\left(\gamma_{k(j)}Q_k-1\right)+\left(\sum_{i=1}^N\gamma_{i(j)}g_i-b_1\right)-\varepsilon\right]^2\\-r_1\sum_{k\in J_C}w_{k(1)}\left[g_1-g_k\right]^2-r_1\sum_{k\in J_{NC}}w_{k(1)}\left[g_1-g_k-Q_k\delta_k\right]^2\\-\varphi_1\left[g_1-\sum_{k\in J_{NC}}\delta_k\right]^2.\end{gathered}$$

If the director does not communicate his signal, then it can be similarly shown that his payoff upon non-communication, U_1^{NC}, is equal to

$$\begin{gathered}-\sum_{j=1}^m q_j\left[\delta_1\left(\gamma_{1(j)}Q_1-1\right)+\sum_{k\in J_{NC}}\delta_k\left(\gamma_{k(j)}Q_k-1\right)+\left(\sum_{i=1}^N\gamma_{i(j)}g_i-b_1\right)-\varepsilon\right]^2\\-r_1\sum_{k\in J_C}w_{k(1)}\left[g_1-g_k+\delta_1Q_1\right]^2-r_1\sum_{k\in J_{NC}}w_{k(1)}\left[g_1-g_k+\delta_1Q_1-Q_k\delta_k\right]^2\\-\varphi_1\left[g_1-\sum_{k\in J_{NC}}\delta_k+\delta_1\left(Q_1-1\right)\right]^2.\end{gathered}$$

The director averages these payoffs over all possible values of $x_2,...,x_N,\varepsilon$ and chooses to communicate his signal if and only if

$$\int U_1^C f_2...f_N f_\varepsilon dx_2...dx_N d\varepsilon > c_1+\int U_1^{NC} f_2...f_N f_\varepsilon dx_2...dx_N d\varepsilon. \tag{B4}$$

If we open the brackets in U_1^C and U_1^{NC}, it is easy to see that the expressions inside the integrals are some linear combinations of quadratic terms δ_i^2,ε^2, interaction terms $\delta_i\delta_j,\delta_i\varepsilon$, linear terms δ_i,ε, and a constant. Note also that the signal of director $k, k\neq 1$ enters U_1^C and U_1^{NC} with a non-zero coefficient only if $x_k\in NC_k$. Also, because $\delta_i=x_i-\mathbb{E}\left[x_i|x_i\in NC_i\right]$, then

$$\int_{NC_i}\delta_i f_i\left(x_i\right)dx_i=0.$$

It follows that all linear terms for $\delta_i, i\geq 2$, all interaction terms $\delta_i\delta_j, i\geq 2$, and all terms including ε, on both sides of $(B4)$ integrate to zero. Hence, only quadratic terms ε^2,δ_i^2, $i\in J_{NC}\cup\{1\}$, the linear term δ_1, and the constant remain. Note also that the constant terms and the coefficients for terms ε^2 and $\delta_i^2, i\in J_{NC}$ in both U_1^C and U_1^{NC} are the same. Besides, the integral over δ_i^2 is taken over the same set NC_i on both sides of $(B4)$. Hence, the integrals over terms ε^2 and $\delta_i^2, i\in J_{NC}$ on both sides of $(B4)$ cancel out. Finally, δ_1^2 and δ_1 do not enter the expression for U_1^C and only enter U_1^{NC}. The coefficient for δ_1^2 in the expression for U_1^{NC} is equal to $-A_1$, and the coefficient for δ_1 is equal to $2B_1$, where $A_1>0$ and B_1 are given in the statement of the lemma. Hence, $(B4)$ is equivalent to $A_1\delta_1^2-2B_1\delta_1-c_1>0$, which proves the lemma.

Proof of Lemma C.3. Denote $\delta_i=x_i-y_i$. For any given realization of $x_1,...,x_N,\varepsilon$, suppose that signals $x_i, i\in J_C$ are communicated in equilibrium and signals $x_i, i\in J_{NC}$ are not communicated. Using the derivations in the proof of Lemma C.2, firm value satisfies

$$V\left(x_1,...,x_N,\varepsilon\right)=V_0-\sum_{j=1}^m q_j\left[\sum_{k\in J_{NC}}\delta_k\left(\gamma_{k(j)}Q_k-1\right)+\sum_{i=1}^N\gamma_{i(j)}g_i-\varepsilon\right]^2,$$

and expected firm value is

$$\mathbb{E}(V) = \int [\ V(x_1, ..., x_N, \varepsilon)]\ f_1(x_1) ... f_N(x_N)\ f_\varepsilon(\varepsilon)\ dx_1 ... dx_N d\varepsilon.$$

By the same argument as in the proof of Lemma C.2, the integral over all linear terms δ_i and interaction terms $\delta_i\delta_j$ is equal to 0. Also, because all quadratic terms δ_i^2 enter additively, the integral over these terms is equal to the sum of the corresponding integrals for individual signals. The coefficient before δ_i^2 for $i \in J_C$ is 0. Finally, note that $i \in J_{NC}$ if and only if $x_i \notin C_i$. Integrating over all possible realizations of $x_1, ..., x_N, \varepsilon$, we get the expression in the statement of the lemma.

Proof of Lemma C.4. Since firm value is $V_0 - (h(a_1, ..., a_N) - \sum_{i=1}^N x_i - \varepsilon)^2$, then, according to Definition 1, a decision rule efficiently aggregates information if and only if expected firm value is $V_0 - \mathbb{E}\varepsilon^2$. The absence of communication means that the communication interval $C_i = \emptyset$, and hence $y_i = \mathbb{E}[x_i | x_i \notin C_i] = 0$. According to Lemma C.3, when $b_i = r_i = \varphi_i = 0$, expected firm value is given by

$$\mathbb{E}[V] = V_0 - \mathbb{E}\varepsilon^2 - \sum_{k=1}^N \left[\sum_{j=1}^m q_j \left(\gamma_{k(j)} Q_k - 1\ \right)^2 \right] \int x_k^2 f_k(x_k)\ dx_k,$$

where $Q_i = \sum_j q_j \gamma_{i(j)} (\sum_j q_j \gamma_{i(j)}^2)^{-1}$. Hence, expected firm value equals $V_0 - \mathbb{E}\varepsilon^2$ if and only if

$$\sum_{j=1}^m q_j \left(\gamma_{k(j)} Q_k - 1\ \right)^2 = 0$$

for all k. Hence, for any j such that $q_j > 0$, it must be that $\gamma_{k(j)} Q_k = 1$. It follows that if $q_{j_1} > 0$ and $q_{j_2} > 0$, then $\gamma_{k(j_1)} = \gamma_{k(j_2)}$, i.e., the coefficient on a_k is the same for the two linear combinations. Hence, any such decision rule is equivalent to a deterministic decision rule, where $q_1 = 1$ and $q_j = 0$ for $j > 1$. The proof for other parameters of directors' preferences is similar.

Proof of Proposition C.1.

(1) If $b_k = \varphi_k = 0$ for all k, then $B_k = 0$ in the statement of Lemma C.2 and hence there exists an equilibrium in which director k communicates his signal x_k if and only if $|x_k| > d_k$, where

$$d_k = \left(\frac{c_k}{A_k}\right)^{1/2} = \left(\frac{c_k}{\sum_{j=1}^m q_j \left[1 - \gamma_{k(j)} Q_k\ \right]^2 + r_k Q_k^2}\right)^{1/2}.$$

Note that Q_k, and hence d_k, only depend on r_k. Hence, expected firm value only depends on r_1 through the following term

$$V(r_1) = - \left[\sum_{j=1}^m q_j \left(\gamma_{1(j)} Q_1 - 1\ \right)^2 \right] \int_{-d_1}^{d_1} x^2 f_1(x)\ dx = -V_1(r_1)\, V_2(r_1).$$

It is straightforward to show that $\lim_{r_1\to 0+} V_1'(r_1) = 0$ and thus,

$$\begin{aligned}
\lim_{r_1\to 0+} V'(r_1) &= -\lim_{r_1\to 0+} V_1(r_1) \lim_{r_1\to 0+} V_2'(r_1) \\
&= \lim_{r_1\to 0+} \left[\sum_{j=1}^{m} q_j \left(\gamma_{1(j)} \frac{\sum_{k=1}^{m} q_k \gamma_{1(k)}}{\sum_{k=1}^{m} q_k \gamma_{1(k)}^2} - 1\right)^2\right] c_1^{3/2} \lim_{r_1\to 0+} Q_1^2 f_1(d_1) \lim_{r_1\to 0+} (A_1)^{-5/2} \\
&= c_1^{3/2} \lim_{r_1\to 0+} Q_1^2 \lim_{r_1\to 0+} f_1(d_1) \left[\lim_{r_1\to 0+} (A_1)\right]^{-3/2}.
\end{aligned}$$

Finally, if $\gamma_{1(j)} \neq \gamma_{1(j')}$, then $\lim_{r_1\to 0+} A_1 > 0$. Indeed,

$$\lim_{r_1\to 0+} A_1 = \sum_{j=1}^{m} q_j \left[1 - \gamma_{1(j)} \frac{\sum_k q_k \gamma_{1(k)}}{\sum_k q_k \gamma_{1(k)}^2}\right]^2.$$

Each term is non-negative and hence the sum can only equal zero if $\gamma_{1(j)} \frac{\sum_k q_k \gamma_{1(k)}}{\sum_k q_k \gamma_{1(k)}^2}$ equals 1 for all j. However, this contradicts the fact that $\gamma_{1(j)} \neq \gamma_{1(j')}$ for some j, j'. Thus, $\lim_{r_1\to 0+} V'(r_1)$ is strictly positive and hence expected firm value is maximized at $r_1^* > 0$.

Proofs of Section III

Proof of Proposition D1.

1. Suppose no signal was revealed. We verify that voting for the proposal if and only if $x_1 > 0$ is indeed the best response strategy of director 1 given that director 2 follows this strategy. Since director 2 did not reveal his signal, director 1 infers that x_2 is uniformly distributed on $[-d, d]$. Then, the utility of director 1 from voting for and against the proposal is, respectively:

$$\begin{aligned}
U_1(a_1 = 1) &= -\int_{-d}^{0} \left[\tfrac{1}{2}(x_1 + x_2 - 1)^2 + \tfrac{1}{2}(x_1 + x_2 + 1)^2 + r\right] \tfrac{1}{2d} dx_2 - \int_0^d \left[(x_1 + x_2 - 1)^2\right] \tfrac{1}{2d} dx_2, \\
U_1(a_1 = -1) &= -\int_{-d}^{0} \left[(x_1 + x_2 + 1)^2\right] \tfrac{1}{2d} dx_2 - \int_0^d \left[\tfrac{1}{2}(x_1 + x_2 - 1)^2 + \tfrac{1}{2}(x_1 + x_2 + 1)^2 + r\right] \tfrac{1}{2d} dx_2.
\end{aligned}$$

and hence the relative benefit of voting for the proposal, $U_1(a_1 = 1) - U_1(a_1 = -1)$, is

$$\begin{gathered}
\int_{-d}^{0} \left[\frac{1}{2}(x_1 + x_2 + 1)^2 - \frac{1}{2}(x_1 + x_2 - 1)^2 - r\right] \frac{1}{2d} dx_2 + \\
\int_0^d \left[-\frac{1}{2}(x_1 + x_2 - 1)^2 + \frac{1}{2}(x_1 + x_2 + 1)^2 + r\right] \frac{1}{2d} dx_2 = 2x_1,
\end{gathered}$$

which is positive if and only if $x_1 > 0$. Hence, it is indeed optimal to vote for the proposal if and only if $x_1 > 0$.

2. Suppose only signal x_1 was revealed. First, suppose $x_1 > 0$ and, according to the conjectured equilibrium, director 1 votes for the proposal. Consider the best response of

director 2.

$$U_2(a_2=1) = -\left[(x_1+x_2-1)^2\right],$$
$$U_2(a_2=-1) = -\left[\frac{1}{2}(x_1+x_2-1)^2+\frac{1}{2}(x_1+x_2+1)^2+r\right],$$

and hence the relative benefit of voting for the proposal, $U_2(a_2=1)-U_2(a_2=-1)$, is positive if and only if

$$-\frac{1}{2}(x_1+x_2-1)^2+\frac{1}{2}(x_1+x_2+1)^2+r>0 \Leftrightarrow x_2>-\frac{r}{2}-x_1,$$

as conjectured. Next, consider the best response of director 1 given that director 2 follows a threshold strategy and votes for the proposal if and only if $x_2>T$. First, if $T\in(-d,d)$, then

$$U_1(a_1=1) = -\int_{-d}^{T}\left[\tfrac{1}{2}(x_1+x_2-1)^2+\tfrac{1}{2}(x_1+x_2+1)^2+r\right]\tfrac{1}{2d}dx_2-\int_{T}^{d}\left[(x_1+x_2-1)^2\right]\tfrac{1}{2d}dx_2,$$
$$U_1(a_1=-1) = -\int_{-d}^{T}\left[(x_1+x_2+1)^2\right]\tfrac{1}{2d}dx_2-\int_{T}^{d}\left[\tfrac{1}{2}(x_1+x_2-1)^2+\tfrac{1}{2}(x_1+x_2+1)^2+r\right]\tfrac{1}{2d}dx_2,$$

and hence $U_1(a_1=1)-U_1(a_1=-1)$ is given by

$$\int_{-d}^{T}\left[\frac{1}{2}(x_1+x_2+1)^2-\frac{1}{2}(x_1+x_2-1)^2-r\right]\frac{1}{2d}dx_2+$$
$$\int_{T}^{d}\left[-\frac{1}{2}(x_1+x_2-1)^2+\frac{1}{2}(x_1+x_2+1)^2+r\right]\frac{1}{2d}dx_2=2x_1-\frac{rT}{d}.$$

Similarly, if $T<-d$, then $U_1(a_1=1)-U_1(a_1=-1)=2x_1+r$, and if $T>d$, then $U_1(a_1=1)-U_1(a_1=-1)=2x_1-r$. Using these expressions, if director 2 follows a threshold strategy with $T=-\frac{r}{2}-x_1$, then the relative benefit for director 1 from voting for the proposal is either $2x_1+\frac{r}{d}\cdot\left(\frac{r}{2}+x_1\right)$ or $2x_1+r$. Since it is strictly positive for $x_1>0$, this verifies the conjectured equilibrium.

Similarly, if $x_1<0$ and hence director 2 believes that director 1 will vote against the proposal, his relative benefit of voting for the proposal, $U_2(a_2=1)-U_2(a_2=-1)$, is positive if and only if

$$-\frac{1}{2}(x_1+x_2-1)^2+\frac{1}{2}(x_1+x_2+1)^2-r>0 \Leftrightarrow x_2>\frac{r}{2}-x_1,$$

as conjectured. Given this, the relative benefit for director 1 from voting for the proposal is either $2x_1-\frac{r}{d}\cdot\left(\frac{r}{2}-x_1\right)$ or $2x_1-r$. Since it is strictly negative for $x_1<0$, this verifies the conjectured equilibrium.

3. Finally, if both signals were revealed and director 1 believes that director 2 votes for the proposal if and only if $x_1+x_2>0$, the best response for director 1 is to follow exactly the same strategy - it both leads to the highest firm value and allows him to avoid the cost of non-conformity.

Proof of Proposition D2.

Consider director 1 with signal $x_1 > 0$ and his expected utility from revealing and not revealing his signal, given that director 2 follows a threshold communication strategy with threshold d. We will show that it is optimal for director 1 to reveal his signal only if it exceeds some threshold. The proof for the case $x_1 < 0$ is similar, given the symmetry of the setup.

Denote U_1^c (U_1^{nc}) director 1's utility given signals x_1, x_2 if he reveals (does not reveal) his signal, and $\Delta U_1 \equiv U_1^c - U_1^{nc}$. Consider the following four ranges of x_1: $(0, d - \frac{r}{2})$, $(d - \frac{r}{2}, d)$, $(d, d + \frac{r}{2})$, and $(d + \frac{r}{2}, 1)$. If $d - \frac{r}{2} < 0$ or $d + \frac{r}{2} > 1$, some of these ranges may not exist.

Case 1. $0 < x_1 < d - \frac{r}{2}$

If $x_2 < -d$, director 2 reveals his signal. If director 1 reveals his signal, both directors vote against since $x_1 + x_2 < -\frac{r}{2} < 0$ and hence $U_1^c = -(x_1 + x_2 + 1)^2$. If director 1 does not reveal his signal, director 2 votes against and director 1 votes against as well because $x_1 + x_2 < -\frac{r}{2}$. Hence, $U_1^{nc} = -(x_1 + x_2 + 1)^2$. Thus, $\Delta U_1 = 0$.

Similarly, if $x_2 > d$, director 2 reveals his signal. If director 1 reveals his signal, both directors vote for since $x_1 + x_2 > 0$ and hence $U_1^c = -(x_1 + x_2 - 1)^2$. If director 1 does not reveal his signal, director 2 votes for and director 1 votes for as well because $x_1 + x_2 > -\frac{r}{2}$. Hence, $U_1^{nc} = -(x_1 + x_2 - 1)^2$ and $\Delta U_1 = 0$.

Finally, if $-d < x_2 < d$, director 2 does not reveal his signal. If director 1 reveals his signal, he votes for the proposal, and director 2 votes for if $x_2 > -\frac{r}{2} - x_1$, where $0 > -\frac{r}{2} - x_1 > -d$. If director 1 does not reveal his signal, he votes for the proposal, and director 2 votes for if $x_2 > 0$.

Combining these cases, director 1's expected relative benefit of revealing his signal is

$$\begin{aligned}
\mathbb{E}\Delta U_1 \quad -c = \quad & -c - \left[\begin{array}{c} \int_{-d}^{-\frac{r}{2}-x_1} \left[\frac{1}{2}(x_1 + x_2 - 1)^2 + \frac{1}{2}(x_1 + x_2 + 1)^2 + r\right] \frac{1}{2} dx_2 \\ + \int_{-\frac{r}{2}-x_1}^{d} \left[(x_1 + x_2 - 1)^2\right] \frac{1}{2} dx_2 \end{array} \right] \\
& + \left[\begin{array}{c} \int_{-d}^{0} \left[\frac{1}{2}(x_1 + x_2 - 1)^2 + \frac{1}{2}(x_1 + x_2 + 1)^2 + r\right] \frac{1}{2} dx_2 \\ + \int_{0}^{d} \left[(x_1 + x_2 - 1)^2\right] \frac{1}{2} dx_2 \end{array} \right] \\
= \quad & -c + \left(\frac{r}{2} + x_1\right)^2 \frac{1}{2}.
\end{aligned}$$

Case 2. $d - \frac{r}{2} < x_1 < d$

If $x_2 < -d$, director 2 reveals his signal. If director 1 reveals his signal, both directors vote against since $x_1 + x_2 < 0$ and hence $U_1^c = -(x_1 + x_2 + 1)^2$. If director 1 does not reveal his signal, director 2 votes against and director 1 votes for if and only if $x_1 + x_2 > \frac{r}{2} \Leftrightarrow x_2 > -x_1 + \frac{r}{2}$. Since $-x_1 + \frac{r}{2} > -d + \frac{r}{2} > -d$, director 1 always votes against in this range and hence $U_1^{nc} = -(x_1 + x_2 + 1)^2$ and $\Delta U_1 = 0$.

If $x_2 > d$, $\Delta U_1 = 0$ for the same argument as in Case 1.

Finally, if $-d < x_2 < d$, director 2 does not reveal his signal. If director 1 reveals his signal, he votes for the proposal, and director 2 votes for if $x_2 > -\frac{r}{2} - x_1$. Since $-\frac{r}{2} - x_1 < -d < x_2$, director 2 always votes for in this range and hence $U_1^c = -(x_1 + x_2 - 1)^2$. If director 1 does not reveal his signal, he votes for the proposal, and director 2 votes for if $x_2 > 0$. Hence, $U_1^{nc} = -(x_1+x_2-1)^2$, $\Delta U_1 = 0$ for $x_2 > 0$, and $U_1^{nc} = -\frac{1}{2}(x_1+x_2+1)^2 - \frac{1}{2}(x_1+x_2-1)^2 - r$, $\Delta U_1 = \frac{1}{2}(x_1+x_2+1)^2 - \frac{1}{2}(x_1+x_2-1)^2 + r$ for $x_2 < 0$.

Combining these ranges of x_2, director 1's expected relative benefit of revealing his signal is

$$\begin{aligned}\mathbb{E}\Delta U_1 - c &= -c + \int_{-d}^{0} \left[\frac{1}{2}(x_1+x_2+1)^2 - \frac{1}{2}(x_1+x_2-1)^2 + r\right]\frac{1}{2}dx_2 \\ &= -c + \left[(2x_1+r)d - d^2\right]\frac{1}{2}.\end{aligned}$$

Case 3. $d < x_1 < d + \frac{r}{2}$

If $x_2 < -d$, director 2 reveals his signal. If director 1 reveals his signal, directors vote for if and only if $x_2 > -x_1$, where $-x_1 < -d$. Hence, for $x_2 \in (-1, -x_1)$, $U_1^c = -(x_1+x_2+1)^2$ and for $x_2 \in (-x_1, -d)$, $U_1^c = -(x_1+x_2-1)^2$. If director 1 does not reveal his signal, director 2 votes against and director 1 votes for if and only if $x_2 > -x_1 + \frac{r}{2}$. Since $-x_1 + \frac{r}{2} > -d - \frac{r}{2} + \frac{r}{2} = -d$, director 1 always votes against in this range and hence $U_1^{nc} = -(x_1+x_2+1)^2$. Thus, $\Delta U_1 = 0$ for $x_2 \in (-1, -x_1)$ and $\Delta U_1 = (x_1+x_2+1)^2 - (x_1+x_2-1)^2$ for $x_2 \in (-x_1, -d)$.

If $x_2 > d$, $\Delta U_1 = 0$ for the same argument as in Case 1. Finally, repeating the argument in Case 2, if $-d < x_2 < d$, ΔU_1 is the same as in Case 2.

Combining these ranges of x_2, director 1's expected relative benefit of revealing his signal is

$$\begin{aligned}\mathbb{E}\Delta U_1 - c &= -c + \int_{-x_1}^{-d} \left[(x_1+x_2+1)^2 - (x_1+x_2-1)^2\right]\frac{1}{2}dx_2 \\ &\quad + \int_{-d}^{0} \left[\frac{1}{2}(x_1+x_2+1)^2 - \frac{1}{2}(x_1+x_2-1)^2 + r\right]\frac{1}{2}dx_2 \\ &= -c + \left[2x_1^2 + d^2 - 2x_1 d + rd\right]\frac{1}{2}.\end{aligned}$$

Case 4. $d + \frac{r}{2} < x_1 < 1$

If $x_2 < -d$, director 2 reveals his signal. If director 1 reveals his signal, directors vote for if and only if $x_2 > -x_1$, where $-x_1 < -d$. Hence, for $x_2 \in (-1, -x_1)$, $U_1^c = -(x_1+x_2+1)^2$ and for $x_2 \in (-x_1, -d)$, $U_1^c = -(x_1+x_2-1)^2$. If director 1 does not reveal his signal, director 2 votes against and director 1 votes for if and only if $x_2 > -x_1 + \frac{r}{2}$. Since $-x_1 + \frac{r}{2} < -d$, then $U_1^{nc} = -(x_1+x_2+1)^2$ for $x_2 \in \left(-1, -x_1 + \frac{r}{2}\right)$ and $U_1^{nc} = -\frac{1}{2}(x_1+x_2+1)^2 - \frac{1}{2}(x_1+x_2-1)^2 - r$ for $x_2 \in \left(-x_1 + \frac{r}{2}, -d\right)$. Thus, $\Delta U_1 = 0$ for $x_2 \in (-1, -x_1)$, $\Delta U_1 = (x_1+x_2+1)^2 - (x_1+x_2-1)^2$ for $x_2 \in \left(-x_1, -x_1 + \frac{r}{2}\right)$, and $\Delta U_1 = \frac{1}{2}(x_1+x_2+1)^2 - \frac{1}{2}(x_1+x_2-1)^2 + r$ for $x_2 \in \left(-x_1 + \frac{r}{2}, -d\right)$.

If $x_2 > d$, $\Delta U_1 = 0$ for the same argument as in Case 1. Finally, repeating the argument in Case 2, if $-d < x_2 < d$, ΔU_1 is the same as in Case 2.

Combining these cases, director 1's expected relative benefit of revealing his signal is

$$\begin{aligned}
\mathbb{E}\Delta U_1 - c &= -c + \int_{-x_1}^{-x_1+\frac{r}{2}} \left[(x_1 + x_2 + 1)^2 - (x_1 + x_2 - 1)^2 \right] \frac{1}{2} dx_2 \\
&\quad + \int_{-x_1+\frac{r}{2}}^{-d} \left[\frac{1}{2} (x_1 + x_2 + 1)^2 - \frac{1}{2} (x_1 + x_2 - 1)^2 + r \right] \frac{1}{2} dx_2 \\
&\quad + \int_{-d}^{0} \left[\frac{1}{2} (x_1 + x_2 + 1)^2 - \frac{1}{2} (x_1 + x_2 - 1)^2 + r \right] \frac{1}{2} dx_2 \\
&= -c + \frac{1}{2} \left(x_1 r + x_1^2 - \frac{r^2}{4} \right).
\end{aligned}$$

In all the four ranges of x_1, director 1's expected relative benefit of revealing his signal, $\mathbb{E}\Delta U_1 - c$, is strictly increasing in x_1. It is also easy to verify that $\mathbb{E}\Delta U_1 - c$ is continuous at the points $d - \frac{r}{2}$, d, and $d + \frac{r}{2}$ and hence is overall continuous. It follows that the director's best response communication strategy is a threshold one: $\mathbb{E}\Delta U_1 - c > 0$ if and only if x_1 exceeds some threshold, which verifies the conjectured equilibrium.

Finally, to find the equilibrium threshold d, note that $\mathbb{E}\Delta U_1 - c$ must equal zero for $x_1 = d$, which gives the following equation on d :

$$-c + \left[(2d + r) d - d^2 \right] \frac{1}{2} = 0 \Leftrightarrow d^2 + rd - 2c = 0.$$

Since $d > 0$, $d = \frac{-r+\sqrt{r^2+8c}}{2}$, which completes the proof.