For Online Publication: Online Appendix for "Trading and Shareholder Democracy"

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A Extensions of the baseline model

A.1 Social concerns

In this section, we extend the model to study situations in which investors care about the decision on the proposal even if they do not own shares of the company. This may be the case if the firm's decision on the proposal has a social or environmental impact that would affect the investor's utility beyond his share ownership in the firm, e.g., a gun-control issue, pollution, etc. Specifically, we assume that if a shareholder owns $\alpha > 0$ shares in the firm, his utility is given by

$$u(d,\theta,b,\alpha) = \alpha u(d,\theta,b) + \gamma bd \quad (28)$$
$$= \alpha [v_0 + (\theta + b)(d - \phi)] + \gamma bd, \quad (29)$$

where $\gamma \geq 0$ captures the sensitivity of the investor's utility to the proposal beyond his ownership in the firm. The baseline model assumes $\gamma = 0$.

Consider the trading stage, and suppose investors expect the proposal to be accepted if and only if $q > q^*$. Given price p, the shareholder chooses the amount of shares to trade t to solve

$$\max_{t\in[-e,x]} \{(e+t)\, v(b,q^*) + \gamma bH(q^*) - tp\},$$

where $v(b,q^*)$ is given by (7). It follows that the shareholder chooses to buy as many shares as he can if $v(b,q^*) > p$, and sell as many shares as he can if $v(b,q^*) < p$. Recall that $v(b,q^*)$ increases b if and only if $H(q^*) > \phi$. Thus, as in the baseline model, if $H(q^*) > \phi$, then the equilibrium is activist, the shareholder buys x shares if $b > b_a$ and sells e shares if $b < b_a$, where $b_a = G^{-1}(\delta)$. The shareholder with bias b_a is the marginal trader, who is indifferent between buying and selling shares, and the share price is $p_a = v(b_a, q^*)$. Similarly, if $H(q^*) < \phi$, then the equilibrium is conservative. In both cases, since investors are not pivotal for the outcome of the proposal, the trading stage is not directly affected by the proposal's impact on their social attitude.

Consider the voting stage. The buying investors account for the effect of the proposal on their utility beyond its direct impact on the share value. Thus, their as-if-pivotal behavior

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implies that they vote for the proposal if and only if

$$\begin{aligned}\mathbb{E}\left[u\left(1,\theta,b\right)|\text{public signal}\right] &> \mathbb{E}\left[u\left(0,\theta,b\right)|\text{public signal}\right] \Leftrightarrow \\ q+b(1+\frac{\gamma}{e+x}) &> 0 \Leftrightarrow q+b(1+\left(\gamma/e\right)\left(1-\delta\right))>0.\end{aligned}$$

Thus, the proposal is accepted if and only if $q > -q_a(\gamma)$, where

$$q_a(\gamma) \equiv -(1+(\gamma/e)(1-\delta))G^{-1}(1-\tau(1-\delta)). \tag{30}$$

The identity of the marginal voter is the same as in the baseline model and is given by $-q_a(0)$. Recall that $-q_a(0) > b_a$, and notice that $-q_a(\gamma)$ increases in γ if and only if $-q_a(0) > 0$. This implies that $\gamma > 0$ amplifies the incentives of the marginal voter: If $-q_a(0) > 0$, the marginal voter becomes more activist as γ increases, and if $-q_a(0) < 0$, he becomes more conservative, less extreme, and possibly even less activist than the marginal trader if γ is sufficiently large.

Overall, and similar to the baseline model, the equilibrium is the following.

Proposition 9. *An equilibrium of the game with trading and voting always exists.*

(i) An ***activist*** *equilibrium exists if and only if $H(q_a(\gamma)) > \phi$, where*

$$q_a(\gamma) \equiv -(1+(\gamma/e)(1-\delta))G^{-1}(1-\tau(1-\delta)). \tag{31}$$

In this equilibrium, a shareholder with bias b buys x shares if $b > b_a$ and sells his entire endowment e if $b < b_a$, where $b_a \equiv G^{-1}(\delta)$. The proposal is accepted if and only if $q > q_a(\gamma)$, and the share price is given by $p_a = v(b_a, q_a(\gamma))$.

(ii) A ***conservative*** *equilibrium exists if and only if $H(q_c(\gamma)) < \phi$, where*

$$q_c(\gamma) \equiv -(1+(\gamma/e)(1-\delta))G^{-1}((1-\delta)(1-\tau)). \tag{32}$$

In this equilibrium, a shareholder with bias b buys x shares if $b < b_c$ and sells his entire endowment e if $b > b_c$, where $b_c = G^{-1}(1-\delta)$. The proposal is accepted if and only if $q > q_c(\gamma)$, and the share price is given by $p_c = v(b_c, q_c(\gamma))$.

(iii) Other equilibria do not exist.

Since in the activist equilibrium $-q_a(0) > b_a$, larger γ increases the share price in the neighborhood of $\gamma = 0$ if and only if $-q_a(0) < 0$. Indeed, only in those circumstances the distance between $-q_a(\gamma)$ and b_a shrinks. Thus, while the trading phase is not directly affected by γ, it is affected indirectly through it effect on the likelihood of the proposal being accepted by the post-trade shareholder base. Intuitively, although atomistic shareholders do not internalize the effect of their own individual trading decisions on the firm's decision-making, they do take into account how social concerns will affect the marginal voter's behavior, and price it accordingly when trading.

Next, we calculate the expected welfare of the initial shareholder base in the activist equilibrium.

$$\begin{aligned}
W_a &= \Pr[b<b_a]\,\mathbb{E}\left[ep_a+\gamma bH(q_a(\gamma))\,|b<b_a\right] \\
&\quad +\Pr[b>b_a]\,\mathbb{E}\left[(e+x)\,v(b,q_a(\gamma))-xp_a+\gamma bH(q_a(\gamma))\,|b>b_a\right] \\
&= \Pr[b<b_a]\,ep_a+\Pr[b>b_a]\,\mathbb{E}\left[(e+x)\,v(b,q_a(\gamma))-xp_a|b>b_a\right]+\gamma\mathbb{E}[b]\,H(q_a(\gamma)) \\
&= e\cdot v(\beta_a,q_a(\gamma))+\gamma\mathbb{E}[b]\,H(q_a(\gamma)) \\
&= e\cdot(v_0+\beta_a(H(q_a(\gamma))-\phi)+H(q_a(\gamma))\,\mathbb{E}[\theta|q>q_a(\gamma)])+\gamma\mathbb{E}[b]\,H(q_a(\gamma)) \\
&= e\cdot(v_0+(\beta_a+(\gamma/e)\,\mathbb{E}[b])\,(H(q_a(\gamma))-\phi)+H(q_a(\gamma))\,\mathbb{E}[\theta|q>q_a(\gamma)])+\phi\gamma\mathbb{E}[b] \\
&= e\cdot v(\beta_a+(\gamma/e)\,\mathbb{E}[b]\,,q_a(\gamma))+\phi\gamma\mathbb{E}[b]
\end{aligned}$$

Similarly, one can show that the expected welfare of the initial shareholder base in the conservative equilibrium is

$$W_c=e\cdot v(\beta_c+(\gamma/e)\,\mathbb{E}[b]\,,q_c(\gamma))+\phi\gamma\mathbb{E}[b]\,.$$

In both cases, we observe that as γ increases, the shareholder welfare function puts more weight on the unconditional expected bias of the shareholder base $\mathbb{E}[b]$.

For simplicity, we focus on the activist equilibrium. We make several observations. First, if

$$\beta_a+(\gamma/e)\,\mathbb{E}[b]<-q_a(0)\,(1+(\gamma/e)\,(1-\delta))<b_a$$

or

$$\beta_a+(\gamma/e)\,\mathbb{E}[b]>-q_a(0)\,(1+(\gamma/e)\,(1-\delta))>b_a,$$

then we have opposing effects on prices and welfare (the analog of Proposition 5).

Second, notice that in general, $\beta_a+(\gamma/e)\,\mathbb{E}[b]\neq\mathbb{E}[b]$ and $\beta_c+(\gamma/e)\,\mathbb{E}[b]\neq\mathbb{E}[b]$. Thus, the optimal board is biased in this context as well, for the same reasons as in the baseline model. (This is true regardless of whether or not the board's preferences are represented by $v(d,\theta,b_m)$ or $u(d,\theta,b_m)$.)

Last, we consider the effect of liquidity. Notice that δ has the same effect on β_a, b_a, and $-q_a(0)$ as in the baseline model: all of them increase, i.e., become more extreme. However, a larger δ can attenuate the incentives of the marginal voter. Indeed, the term $(1+(\gamma/e)\,(1-\delta))$ decreases in δ. Hence, if $-q_a(0)<0$, then the marginal voter becomes more extreme as δ increases. However, if $-q_a(0)>0$, then the overall effect of δ is ambiguous. Intuitively, higher liquidity implies that the marginal voter has a larger stake in the firm in equilibrium, and hence, puts relatively more weight on the proposal's impact on the share value. This implies that an activist marginal voter ($-q_a(0)>0$) will behave more conservatively, and a conservative marginal voter ($-q_a(0)<0$) will become more activist in relative terms.

A.2 Heterogeneous endowments and trading frictions

In this section, we extend the model to environments in which shareholders differ not only with respect to their attitude toward the proposal, but also with respect to their endowments and ability to trade. Specifically, we assume that a shareholder with bias b has endowment of $e(b) > 0$ and is able to buy not more than $x(b) > 0$ shares. We denote by $\mathbf{e} \equiv \int_{-\bar{b}}^{\bar{b}} e(b)\, dG(b)$ the total endowment. Importantly, we do not impose any restrictions on the functions $e(\cdot)$ and $x(\cdot)$, and as such, we allow for various correlations between the shareholder's attitude toward the proposal, endowment, and ability to trade. In particular, our framework nests cases in which $e(\cdot)$ and $x(\cdot)$ increase in $|b|$, that is, more extreme shareholders have larger endowments and more ability to trade. We demonstrate that our main results continue to hold even when such arbitrary correlations hold.

To begin, note that the functional form of $v(b, q^*)$ does not change and is given by expression (7). Therefore, as in the baseline model, shareholder b buys $x(b)$ shares if $v(b, q^*) > p$ and sells $e(b)$ shares if $v(b, q^*) < p$.

If $H(q^*) > \phi$, then $v(b, q^*)$ increases in b, there exists b_a such that $v(b, q^*) > p \Leftrightarrow b > b_a$, and the equilibrium is activist. The total demand for shares is $D(p) = \int_{b_a}^{\bar{b}} x(b)\, dG(b)$, and the total supply of shares is $S(p) = \int_{-\bar{b}}^{b_a} e(b)\, dG(b)$. The market clears if and only if

$$\int_{b_a}^{\bar{b}} x(b)\, dG(b) = \int_{-\bar{b}}^{b_a} e(b)\, dG(b). \tag{33}$$

It is straightforward to verify that there exists a unique $b_a \in \left(-\bar{b}, \bar{b}\right)$ that solves (33). Similarly, if $H(q^*) < \phi$, then $v(b, q^*)$ decreases in b, there exists b_c such that $v(b, q^*) > p \Leftrightarrow b < b_c$, and the equilibrium is conservative. The total demand for shares is $D(p) = \int_{-\bar{b}}^{b_c} x(b)\, dG(b)$, and the total supply of shares is $S(p) = \int_{b_c}^{\bar{b}} e(b)\, dG(b)$. The market clears if and only if

$$\int_{-\bar{b}}^{b_c} x(b)\, dG(b) = \int_{b_c}^{\bar{b}} e(b)\, dG(b). \tag{34}$$

Again, it is straightforward to verify that there exists a unique $b_c \in \left(-\bar{b}, \bar{b}\right)$ that solves (34).

Notice that if an exogenous shock (weakly) increases $x(b)$ for all b but leaves $e(b)$ unchanged, then the demand for shares increases, the supply does not change, and thus the market can clear only if b_a increases in the activist equilibrium and b_c decreases in the conservative equilibrium. In other words, when trading frictions are relaxed, the marginal trader become more extreme in equilibrium, as in the baseline model.

Next, we analyze the identity of the marginal voter. Notice that Lemma 1 continues to holds in this setup since, as before, each shareholder is more likely to vote for the proposal when q is larger. First, consider the activist equilibrium. Post trade, the shareholder base consists of shareholders with a bias larger than b_a. For any given $b > b_a$, shareholders with bias b collectively hold a fraction

$$g_a(b) \equiv g(b) \frac{x(b) + e(b)}{\mathbf{e}} \tag{35}$$

of all shares. Notice that

$$\begin{aligned}\int_{b_a}^{\bar{b}} g_a(b)\,db &= \frac{1}{\mathbf{e}}\int_{b_a}^{\bar{b}} x(b)\,g(b)\,db + \frac{1}{\mathbf{e}}\int_{b_a}^{\bar{b}} e(b)\,g(b)\,db \\ &= \frac{1}{\mathbf{e}}\int_{-\bar{b}}^{b_a} e(b)\,g(b)\,db + \frac{1}{\mathbf{e}}\int_{b_a}^{\bar{b}} e(b)\,g(b)\,db \\ &= \frac{1}{\mathbf{e}}\int_{-\bar{b}}^{\bar{b}} e(b)\,g(b)\,db = \frac{1}{\mathbf{e}}\mathbf{e} = 1,\end{aligned}$$

where the second equality follows from market clearing. Thus, we can view $g_a(b)$ as a density function with full support on $\left[b_a, \bar{b}\right]$. The corresponding cdf is given by

$$G_a(b) \equiv \int_{b_a}^{b} g_a(b)\,db. \tag{36}$$

Thus, the marginal voter is given by $-q_a \equiv G_a^{-1}(1-\tau)$. Notice that $G_a^{-1}(1-\tau) > b_a$, i.e., the marginal voter is more extreme than the marginal trader, as in the baseline model.

Similarly, in conservative equilibrium, the post-trade shareholder base consists of shareholders with a bias smaller than b_c. For any given $b < b_c$, shareholders with bias b collectively hold a fraction

$$g_c(b) \equiv g(b)\,\frac{x(b)+e(b)}{\mathbf{e}} \tag{37}$$

of all shares. Similar to the activist equilibrium, it can be shown that $\int_{-\bar{b}}^{b_c} g_c(b)\,db = 1$. Thus, we can view $g_c(b)$ as a density function with full support on $\left[-\bar{b}, b_c\right]$, and the corresponding cdf is given by

$$G_c(b) \equiv \int_{-\bar{b}}^{b} g_c(b)\,db. \tag{38}$$

Thus, the marginal voter is given by $-q_c \equiv G_c^{-1}(1-\tau)$, and notice again that $G_c^{-1}(1-\tau) < b_c$, that is, the marginal voter is more extreme than the marginal trader.

Given all of the above, the analysis of Section 4.3 is easily extended to a setup with heterogeneous endowments and trading frictions. In particular, the following result holds.

Proposition 10. *An equilibrium of the game with trading and voting always exists. Let b_a and b_c be the unique solutions of (33) and (34), respectively, and $G_a(b)$ and $G_c(b)$ be the distribution functions as defined by (36) and (38), respectively. Then:*

(*i*) *An **activist** equilibrium exists if and only if $H(q_a) > \phi$, where*

$$q_a \equiv -G_a^{-1}(1-\tau). \tag{39}$$

In this equilibrium, a shareholder with bias b buys $x(b)$ shares if $b > b_a$ and sells his entire endowment $e(b)$ if $b < b_a$. The proposal is accepted if and only if $q > q_a$, and the share price is given by $p_a = v(b_a, q_a)$.

(*ii*) *A* ***conservative*** *equilibrium exists if and only if* $H(q_c) < \phi$, *where*

$$q_c \equiv -G_c^{-1}(1-\tau). \tag{40}$$

In this equilibrium, a shareholder with bias b *buys* $x(b)$ *shares if* $b < b_c$ *and sells his entire endowment* $e(b)$ *if* $b > b_c$. *The proposal is accepted if and only if* $q > q_c$, *and the share price is given by* $p_c = v(b_c, q_c)$.

(*iii*) *Other equilibria do not exist.*

Next, as in the baseline model, we define $\beta_a = \int_{b_a}^{\bar{b}} bg_a(b)\,db$ and $\beta_c = \int_{-\bar{b}}^{b_c} bg_a(b)\,db$ as the average bias of the post-trade shareholder base. Notice that $\beta_a > b_a$ and $\beta_c < b_c$. The expected welfare of the initial shareholder base is

$$\begin{aligned} W_a &= p_a \int_{-\bar{b}}^{b_a} e(b)\,g(b)\,db + \int_{b_a}^{\bar{b}} \left[(e(b)+x(b))\,v(b,q_a) - x(b)\,p_a\right] g(b)\,db \\ &= p_a \left[\int_{-\bar{b}}^{b_a} e(b)\,g(b)\,db - \int_{b_a}^{\bar{b}} x(b)\,g(b)\,db\right] + \int_{b_a}^{\bar{b}} (e(b)+x(b))\,v(b,q_a)\,g(b)\,db \\ &= p_a \cdot 0 + \int_{b_a}^{\bar{b}} (e(b)+x(b))\,v(b,q_a)\,g(b)\,db = \mathbf{e}\int_{b_a}^{\bar{b}} v(b,q_a)\,g_a(b)\,db \\ &= \mathbf{e}\cdot v\left(\int_{b_a}^{\bar{b}} bg_a(b)\,db, q_a\right) = \mathbf{e}\cdot v(\beta_a, q_a), \end{aligned}$$

which is the valuation of the average post-trade shareholder. Indeed, the third equality follows from the market-clearing condition, the fourth equality follows from the definition of $g_a(b) = \frac{e(b)+x(b)}{\mathbf{e}} g(b)$, the fifth equality follows from the linearity of $v(b, q^*)$ in b, and the last equality follows from the definition of β_a. Similar analysis shows that in the conservative equilibrium, $W_c = \mathbf{e}\cdot v(\beta_c, q_c)$.

Since in the activist equilibrium $p_a = v(b_a, q_a)$ and $W_a = \mathbf{e}\cdot v(\beta_a, q_a)$, and in the conservative equilibrium $p_c = v(b_c, q_c)$ and $W_c = \mathbf{e}\cdot v(\beta_c, q_c)$, the analysis of Sections 5 and 6 is extended to the setup with heterogeneous endowment and trading frictions, with the exception of the comparative statics with respect to δ. The analogous analysis about the effect of δ can be obtained if instead one considers an exogenous shock that (weakly) increases $x(b)$ for all b but leaves $e(b)$ unchanged. We omit this analysis for brevity.

A.3 Trading after voting

In this section, we extend the baseline model by allowing for another round of trade after shareholders vote. The purpose of this extension is to explicitly analyze the effect of the voting outcome on the share price and shareholders' welfare, and to demonstrate that prices and welfare can move in opposite directions. In addition, the analysis shows the robustness of our baseline model to a dynamic trading environment.

Suppose that with probability $\varrho \in (0,1)$ there is another round of trade after the voting outcome is determined, but before fundamentals θ are realized. The post-vote trading stage features the same frictions as pre-vote trading, namely, investors can buy at most x shares and cannot sell more than the shares they own at that point (which would be at most $e+x$ if they bought shares at the pre-vote trading stage). With probability $1-\varrho$ trade is not feasible at the post-vote stage, and the game ends. We will focus on the case $\varrho \to 1$. (We introduce $\varrho < 1$ to break a shareholder's indifference in the pre-vote trading stage when such indifference occurs due to the introduction of a second round of trade.)

We denote the price at the second, post-vote, round of trade by $p_{post}(d,q)$, and the price at the first, pre-vote, round of trade by p_{pre}. Note that the post-vote price depends on the voting outcome and the realization of q.

As in the baseline model, we focus on equilibria in which the proposal is accepted if and only if $q > q^*$. Given q^*, the expected shareholder value at the pre-vote stage is

$$\begin{aligned}\pi(b,q^*) &= \varrho\mathbb{E}\left[\max\left\{p_{post}\left(\mathbf{1}_{q>q^*},q\right), v\left(\mathbf{1}_{q>q^*},q,b\right)\right\}\right] + (1-\varrho)\,\mathbb{E}\left[v\left(\mathbf{1}_{q>q^*},q,b\right)\right] \\ &= \varrho\begin{pmatrix} H(q^*)\,\mathbb{E}\left[\max\left\{p_{post}(1,q), v(1,q,b)\right\}|q>q^*\right] \\ +(1-H(q^*))\,\mathbb{E}\left[\max\left\{p_{post}(0,q), v(0,q,b)\right\}|q<q^*\right]\end{pmatrix} + (1-\varrho)\,v(b,q^*),\end{aligned}$$

where $v(d,q,b)$ is given by (1) and $v(b,q^*)$ is given by (7).

Consider the post-vote round of trade, which occurs after the public signal q and the voting outcome d are revealed. Note that

$$v(1,q,b) \geq p_{post}(1,q) \Leftrightarrow b > b_1(q) \equiv \frac{p_{post}(1,q)-v_0}{1-\phi} - q$$

and

$$v(0,q,b) \geq p_{post}(0,q) \Leftrightarrow b < b_0(q) \equiv \frac{p_{post}(0,q)-v_0}{-\phi} - q.$$

In words, buyers upon proposal approval are investors with a large b, and buyers upon proposal rejection are investors with a small b. Moreover, conditional on q and d, prices are determined by the valuation of the marginal trader: $p_{post}(1,q) = v(1,q,b_1(q))$ and $p_{post}(0,q) = v(0,q,b_0(q))$.

Note that b_1 and b_0 do not depend on q. Indeed, suppose $q > q^*$, and let $e_{post}(b) \geq 0$ be the number of shares shareholder b owns at the beginning of the second round of trade. Then, the demand is $x(1-G(b_1(q)))$ and the supply is $\int_{-\bar{b}}^{b_1(q)} e_{post}(b)\,g(b)\,db$. Notice that the demand and supply depend on q only through $b_1(q)$, and thus, market clearing which uniquely pins down $b_1(q)$, implies that $b_1(q)$ does not depend on q. Similarly, $b_0(q)$ does not depend on q.

Given the observations above, we have

$$\begin{aligned}
\pi(b,q^*) &= \varrho\begin{pmatrix} H(q^*)\,\mathbb{E}\left[\max\{v(1,q,b_1),v(1,q,b)\}\,|q>q^*\right] \\ +(1-H(q^*))\,\mathbb{E}\left[\max\{v(0,q,b_0),v(0,q,b)\}\,|q<q^*\right]\end{pmatrix}+(1-\varrho)\,v(b,q^*) \\
&= \varrho\left(v_0+\begin{array}{c}(1-\phi)\,H(q^*)\,\mathbb{E}\left[\max\{q+b_1,q+b\}\,|q>q^*\right] \\ -\phi\,(1-H(q^*))\,\mathbb{E}\left[\min\{q+b_0,q+b\}\,|q<q^*\right]\end{array}\right)+(1-\varrho)\,v(b,q^*) \\
&= v_0+H(q^*)\,\mathbb{E}\left[q|q>q^*\right]+\varrho\begin{pmatrix}(1-\phi)\,H(q^*)\max\{b_1,b\} \\ -\phi\,(1-H(q^*))\min\{b_0,b\}\end{pmatrix}+(1-\varrho)\,b\,(H(q^*)-\phi)\,.
\end{aligned}$$

Recall that we focus on the case $\varrho\to 1$. Therefore, (1) if $b<\min\{b_0,b_1\}$ then $\pi(b,q^*)$ (weakly) decreases in b; (2) if $b>\max\{b_0,b_1\}$ then $\pi(b,q^*)$ (weakly) increases in b; and (3) if $\min\{b_0,b_1\}<b<\max\{b_0,b_1\}$ then $\pi(b,q^*)$ (weakly) increases in b if and only if $H(q^*)-\phi>0$. We conclude that in equilibrium of the first round of trade, there exist b_L^* and b_H^* , $-\bar{b}\le b_L^*<b_H^*\le\bar{b}$, such that the investor buys shares if and only if $b\le b_L^*$ or $b\ge b_H^*$. Thus, market clearing in the first round of trade implies that

$$[G(b_L^*)+1-G(b_H^*)]\,x=[G(b_H^*)-G(b_L^*)]\,e\Leftrightarrow G(b_H^*)-G(b_L^*)=\delta, \tag{41}$$

where

$$p_{pre}=\pi(b_H^*,q^*)=\pi(b_L^*,q^*)\,. \tag{42}$$

Since $\pi(\cdot,q^*)$ has a unique minimum and is convex, there are unique b_L^* and b_H^* , $-\bar{b}\le b_L^*<b_H^*\le\bar{b}$, that solve (41) and (42). Notice that b_L^* and b_H^* depend on b_0 and b_1, where either $b_L^*>-\bar{b}$ or $b_H^*<\bar{b}$, but not necessarily both.

As in the baseline model, if the cutoff q^* is taken as given, we have two separate cases to analyze:

1. Suppose $H(q^*)>\phi$. Then, the minimum of $\pi(b,q^*)$ is obtained at $\min\{b_0,b_1\}$ and thus, $b_L^*<\min\{b_0,b_1\}<b_H^*$. Thus, if $q>q^*$, the demand is $(1-G(b_1))\,x$, the supply is $[G(b_L^*)+\max\{0,G(b_1)-G(b_H^*)\}]\,(e+x)$, and the market clears whenever

$$(1-G(b_1))\,\delta=G(b_L^*)+\max\{0,G(b_1)-G(b_H^*)\}. \tag{43}$$

   If $q<q^*$, the demand is $G(b_0)\,x$, the supply is $\min\{1-G(b_0),1-G(b_H^*)\}\,(e+x)$, and the market clears whenever

$$G(b_0)\,\delta=\min\{1-G(b_0),1-G(b_H^*)\}. \tag{44}$$

   Combined with (41) and (42), equations (43) and (44) allow us to pin down b_0 and b_1 in the case where $H(q^*)>\phi$.

2. Suppose $H(q^*)<\phi$. Then, the minimum of $\pi(b,q^*)$ is obtained at $\max\{b_0,b_1\}$ and thus, $b_L^*<\max\{b_0,b_1\}<b_H^*$. Thus, if $q>q^*$, the demand is $(1-G(b_1))\,x$, the supply is

$\min\{G(b_1), G(b_L^*)\}(e+x)$, and the market clears whenever

$$(1-G(b_1))\,\delta = \min\{G(b_1), G(b_L^*)\}. \tag{45}$$

If $q<q^*$, the demand is $G(b_0)\,x$, the supply is $[1-G(b_H^*)+\max\{0, G(b_L^*)-G(b_0)\}](e+x)$, and the market clears whenever

$$G(b_0)\,\delta = 1-G(b_H^*)+\max\{0, G(b_L^*)-G(b_0)\}. \tag{46}$$

Combined with (41) and (42), equations (45) and (46) allow us to pin down b_0 and b_1 in the case where $H(q^*)<\phi$.

Essentially, unlike the baseline model, it is now possible that both very conservative and very activist investors buy shares at the first-round of stage, while moderate investors sell. Then, at the post-vote trading stage, the activist shareholders buy shares from (sell shares to) the conservative shareholders if the proposal is accepted (rejected). Intuitively, even if the proposal is likely to be accepted, the option to trade after the vote gives incentives to conservative shareholders to buy shares with anticipation that they can sell it to activists if the expected outcome materializes; however, they can also enjoy the upside of owning many shares in the firm if the proposal is unexpectedly rejected.

The analysis so far has taken q^* as given. Below, we endogenize the voting stage and analyze of the extended game with post-vote trading for the case $\phi=0$. We fully characterize the equilibrium and illustrate the opposing welfare and price reactions to the voting outcome.

A.3.1 Complete analysis for $\phi = 0$

If $\phi=0$, then $v(d,\theta,b)=v_0+(\theta+b)\,d$ and

$$\begin{aligned}\pi(b,q^*) &= v_0+H(q^*)\,E[q|q>q^*]+[\varrho\max\{b_1,b\}+(1-\varrho)\,b]\,H(q^*)\\ &= v(\varrho\max\{b_1,b\}+(1-\varrho)\,b,q^*)\end{aligned}$$

is strictly increasing in b. Therefore, $b_L^*=-\bar{b}$, and shareholders buy shares at the pre-vote trading stage if and only if $b>b_H^*=b_a=G^{-1}(\delta)$, just as in the baseline model. Suppose a second round of trade after the voting stage occurs. If $q<q^*$, then the proposal is rejected, the share value to all shareholders is v_0, and no trade occurs. Suppose $q>q^*$. Equation (43) implies that b_1 solves

$$(1-G(b_1))\,\delta = \max\{0, G(b_1)-\delta\}.$$

If $b_1\le G^{-1}(\delta)$, then the supply is zero, and hence, the market cannot clear. Thus, it must be that $b_1>G^{-1}(\delta)$, and in particular,

$$b_1 = G^{-1}(\frac{2\delta}{1+\delta}),$$

which is indeed larger than $G^{-1}(\delta)$. In equilibrium: (1) shareholders with $b<b_a=G^{-1}(\delta)$ sell; (2) shareholders with $b\in(b_a,b_1)=(G^{-1}(\delta), G^{-1}(\frac{2\delta}{1+\delta}))$ buy at the first stage and sell

at the second stage if the proposal is approved and there is trade; and (3) shareholders with $b > b_1 = G^{-1}(\frac{2\delta}{1+\delta})$ buy at the first stage and buy more shares at the second stage if the proposal is approved and there is trade. The share price in the first round of trade (assuming $\varrho \to 1$) is the valuation of the marginal trader b_a, which is given by $\pi(b_a, q^*)$. Therefore,

$$p_{pre} = \pi(b_a, q^*) = v(\max\{b_1, b_a\}, q^*) = v(b_1, q^*) = v_0 + H(q^*)(b_1 + E[q|q > q^*]).$$

In the second round of trade, the share price upon rejection of the proposal is always v_0. The share price upon approval of the proposal and realization of q is the valuation of the marginal trader b_1, and hence, $p_{post}(1, q) = v(1, q, b_1)$. Thus, $p_{post}(d, q) = v(d, q, b_1)$.

Last, we consider the identity of the marginal voter. At the voting stage, the shareholders of the firm are those with $b > b_a$, and each owns $x + e$ shares. A shareholder with $b \in [b_a, b_1]$ expects to get v_0 if the proposal is rejected and $v_0 + q + b_1$ if the proposal is accepted. Thus, this shareholder votes for the proposal if and only if $q > -b_1$. A shareholder with $b > b_1$ expects to get $(e + x)v_0$ if the proposal is rejected and $(e + x)(v_0 + b + q) + x(b - b_1)$ if the proposal is accepted. Thus, this shareholder votes for the proposal if and only if $q > -[b(1 + \delta) - \delta b_1]$. Combined, shareholders vote for the proposal if and only if

$$q > -[\max\{b, b_1\}(1 + \delta) - \delta b_1]. \tag{47}$$

Notice that $b(1 + \delta) - \delta b_1 > b \Leftrightarrow b > b_1$, and thus the introduction of the post-vote trading stage implies that shareholders vote as if their bias is higher (whether or not they intend to sell or buy their shares). Intuitively, shareholders who expect to sell have stronger incentives to approve the proposal because they internalize the positive effect of approving the proposal on the valuation of the marginal trader at the post-vote stage, b_1, and therefore, on the price they expect to receive for their shares. The buying shareholders also have stronger incentives to approve the proposal since they benefit when the proposal is approved.

As in the baseline model, the marginal voter is still given by the shareholder with bias $G^{-1}(1 - (1 - \delta)\tau)$. However, the proposal is approved if and only if

$$q > q_a = -\left[\max\left\{G^{-1}(1 - (1 - \delta)\tau), b_1\right\}(1 + \delta) - \delta b_1\right].$$

Notice that $G^{-1}(1 - (1 - \delta)\tau) < b_1 \Leftrightarrow \tau \geq \frac{1}{1+\delta}$. Thus, if $\tau \geq \frac{1}{1+\delta}$, then the marginal voter is among shareholders with $b \in [b_a, b_1]$, and the proposal is approved if and only if $q > -b_1$. If $\tau < \frac{1}{1+\delta}$, then the marginal voter is among shareholders with $b > b_1$, and the proposal is approved if and only if $q > -[G^{-1}(1 - (1 - \delta)\tau)(1 + \delta) - \delta b_1]$. Either way, the proposal is approved only if $q > -b_1$.

The next result summarizes the observations above.

Proposition 11. *Consider the game with two rounds of trade, one before the vote and one after, and suppose $\phi = 0$. Define*

$$q_a \equiv -\left[\max\left\{G^{-1}(1 - (1 - \delta)\tau), b_1\right\}(1 + \delta) - \delta b_1\right], \tag{48}$$

$$b_a \equiv G^{-1}(\delta), \text{ and } b_1 \equiv G^{-1}(\frac{2\delta}{1+\delta}). \tag{49}$$

Then, the unique equilibrium is activist and has the following properties:

(i) *In the first round of trade, a shareholder with bias b buys x shares if $b > b_a$ and sells his entire endowment e if $b < b_a$. The share price at the first round of trade is given by $p_{pre} = v(b_1, q_a)$.*

(ii) *At the voting stage, a shareholder votes for the proposal if and only if*

$$q > -[\max\{b, b_1\}(1+\delta) - \delta b_1], \tag{50}$$

and the proposal is accepted if and only if $q > q_a$.

(iii) *In the second round of trade, no trade occurs if the proposal is rejected. If the proposal is accepted, a shareholder with bias b buys x shares if $b > b_1$ and sells his entire holdings $e+x$ if $b \in [b_a, b_1]$. The share price at the second round of trade conditional on realization q and decision d is given by $p_{post}(d,q) = v(d,q,b_1)$.*

Price and welfare reactions to voting outcomes. Given the result above, we can analyze the average price reaction to the voting outcome, defined as

$$\Delta P(d) \equiv \mathbb{E}[p_{post}(d,q) - p_{pre}|d].$$

Since prices are martingales, $\Pr[d=1]\Delta P(d) + \Pr[d=0]\Delta P(0) = 0$, and thus, $\Delta P(1) > 0 \Leftrightarrow \Delta P(0) < 0$. Also notice that

$$\begin{aligned}
\Delta P(1) &= \mathbb{E}[p_{post}(1,q) - p_{pre}|q > q_a] = \mathbb{E}[v(1,q,b_1)|q > q_a] - v(b_1, q_a) \\
&= \mathbb{E}[v_0 + q + b_1|q > q_a] - v_0 - b_1 H(q_a) - H(q_a)\mathbb{E}[\theta|q > q_a] \\
&= (1 - H(q_a))(\mathbb{E}[\theta|q > q_a] + b_1).
\end{aligned}$$

Thus,

$$\Delta P(1) < 0 \Leftrightarrow -\mathbb{E}[\theta|q > q_a] > b_1.$$

Intuitively, the average price reaction depends on how the proposal approval, on average, affects the payoff of the marginal trader b_1 at the post-vote trading stage.

Next, we turn to calculate the reaction of shareholder welfare to the voting outcome (when $\varrho \to 1$). Recall that shareholders with $b_a < b < b_1$ buy x shares in the first round of trade, and then sell their entire stake for $p_{post}(1,q)$ after the vote if the proposal is approved, and get v_0 if it is rejected. Thus, their valuation is the same as that of a shareholder with bias b_1. Given this observation, the expected welfare of the initial shareholder base (prior to the first round of trade) is given by

$$
\begin{aligned}
W &= \Pr\left[b<b_a\right] ep_{pre} + \Pr\left[b_a<b<b_1\right]\mathbb{E}\left[(e+x)\,v\left(b_1,q_a\right) - xp_{pre}|b_a<b<b_1\right] \\
&\quad + \Pr\left[b>b_1\right]\mathbb{E}\left[(e+x)\,v\left(b,q_a\right) - xp_{pre} + x\left(v\left(b,q_a\right) - v\left(b_1,q_a\right)\right)|b>b_1\right] \\
&= \Pr\left[b_a<b<b_1\right](e+x)\,v\left(b_1,q_a\right) \\
&\quad + \Pr\left[b>b_1\right]\mathbb{E}\left[(e+x)\,v\left(b,q_a\right) + x\left(v\left(b,q_a\right) - v\left(b_1,q_a\right)\right)|b>b_1\right] \\
&= \Pr\left[b>b_1\right](e+2x)\,\mathbb{E}\left[v\left(b,q_a\right)|b>b_1\right] = e\cdot\mathbb{E}\left[v\left(b,q_a\right)|b>b_1\right] \\
&= e\cdot v\left(\mathbb{E}\left[b|b>b_1\right],q_a\right).
\end{aligned}
$$

The second equality follows from market clearing at the pre-vote stage, the third equality follows from market clearing at the post-vote stage, the fourth equality follows from the definition of b_1, and the last equality follows from the linearity of $v\left(\cdot,q_a\right)$. Thus, as in the baseline model, the expected welfare is given by the valuation of the average post-trade shareholder, but now it accounts for both the pre-vote and the post-vote trading stages.

The expected average welfare of shareholders conditional on voting outcome d is given by $e\cdot v_0$ if $d=0$ and by

$$W(1) = e\cdot\left(v_0 + \mathbb{E}\left[b|b>b_1\right] + \mathbb{E}\left[\theta|q>q_a\right]\right)$$

if $d=1$. As with prices, we can define the average welfare reaction to the voting outcome as

$$\Delta W(d) \equiv \mathbb{E}\left[W(d) - W|d\right].$$

Notice that $\Delta W(1)>0 \Leftrightarrow \Delta W(0)<0$. Moreover,

$$
\begin{aligned}
\Delta W(1) &= e\cdot\left(v_0 + \mathbb{E}\left[b|b>b_1\right] + \mathbb{E}\left[\theta|q>q_a\right]\right) - e\cdot v\left(\mathbb{E}\left[b|b>b_1\right],q_a\right) \\
&= e\left(1-H\left(q_a\right)\right)\cdot\left(\mathbb{E}\left[b|b>b_1\right] + \mathbb{E}\left[\theta|q>q_a\right]\right),
\end{aligned}
$$

and thus,

$$\Delta W(1)>0 \Leftrightarrow \mathbb{E}\left[b|b>b_1\right] > -\mathbb{E}\left[\theta|q>q_a\right].$$

Since $\mathbb{E}\left[b|b>b_1\right]>b_1$, the next proposition follows.

Proposition 12. *The average welfare and price reactions to voting outcomes have opposite signs if and only if*

$$\mathbb{E}\left[b|b>b_1\right] > -\mathbb{E}\left[\theta|q>q_a\right] > b_1. \tag{51}$$

In those cases, the average welfare reaction to proposal approval (rejection) is positive (negative), while the average price reaction is negative (positive).

When is condition (51) satisfied? Recall that $q_a\le -b_1$, where the inequality is strict if and only if $\tau<\frac{1}{1+\delta}$. Thus, if $\tau\ge\frac{1}{1+\delta}$, then $q_a=-b_1$, $-\mathbb{E}\left[\theta|q>q_a\right] = -\mathbb{E}\left[\theta|q>-b_1\right] < b_1$, and condition (51) does not hold. Intuitively, if the marginal voter behaves the same way as the marginal trader in post-vote trading, then the price reaction to proposal approval must be positive since the approval always benefits the marginal trader who sets the post-vote price. Since shareholder welfare is determined by the post-trade shareholder, who is even more activist than the marginal trader and hence more biased toward the proposal, the average welfare reaction is also positive. In this case, both price and welfare reactions to proposal approval are positive.

In contrast, if $\tau < \frac{1}{1+\delta}$, then $q_a < -b_1$ and the marginal voter is more activist than the marginal trader. Therefore, whenever $q \in [q_a, -b_1]$, the proposal is accepted although it reduces the value of the marginal trader. In those cases, the price reaction to proposal approval is negative. At the same time, $\mathbb{E}[b|b > b_1] > b_1$ implies that it is possible that the average post-trade shareholder is more activist than the marginal voter, i.e., $\mathbb{E}[b|b > b_1] > -q_a$. If in addition $q \in [q_a, -b_1]$, then approval of the proposal benefits the average post-trade shareholder, and as a result the realized welfare reaction is positive. If the weight on realizations of $q \in [q_a, -b_1]$ is sufficiently high, which certain distributions can guarantee, then the welfare and price reaction to voting outcomes have opposite signs.

A.4 Trading with partial sales of endowments

Our basic model assumes that shareholders can sell their entire endowment. In this section, we relax this assumption to incorporate scenarios where trading frictions are particularly high and do not allow initial shareholders to exit the firm completely. Specifically, we assume that when trading, shareholders can buy up to x shares or sell up to $y \in (0, e)$ shares, while retaining the remaining $e - y$ shares. All other assumptions in the baseline model remain unchanged. We provide a general discussion of the model in which we allow for $y < e$ in Section A.4.1, offer a more rigorous analysis in Section A.4.2, and gather the formal proofs in Section A.4.3.

A.4.1 Discussion of the model with $y < e$

Note that this extension allows us to separate the effect of market depth (captured by x and y, the amounts that shareholders can trade) from the effects of $\frac{x}{y}$, which captures the asymmetry between trading frictionson the buy-side and those on the sell-side. For simplicity, in what follows, we set $e = 1$. The formal analysis of this extension is presented in Section A.4.2, and we only summarize the key steps and conclusions here. As the following discussion demonstrates, our main results continue to hold in this extension.

When shareholders cannot exit their entire position in the firm, the post-trade shareholder base is composed of the buying shareholders, who hold $1 + x$ shares each, and the selling shareholders, who hold $1 - y$ shares each. This change does not materially affect the characterization of the equilibrium as given in Proposition 3. In particular, any equilibrium is either conservative or activist. For example, if the equilibrium is conservative, the marginal voter is given by

$$-q_c = \begin{cases} G^{-1}\left(\frac{1-\tau}{1+\frac{\delta}{1-\delta}y}\right) & \text{if } \frac{x(1-y)}{x+y} \leq \tau \\ G^{-1}(1 - \frac{\tau}{1-y}) & \text{if } \frac{x(1-y)}{x+y} > \tau \end{cases} \tag{52}$$

and the marginal trader is given by

$$b_c = G^{-1}(1 - \frac{x}{x+y}). \tag{53}$$

In this equilibrium, which exists if and only if $q_c > F^{-1}(1-\phi)$, the shareholder buys x shares if $b < b_c$ and sells y shares otherwise. The proposal is accepted if and only if $q > q_c$, and the share price is given by $p_c = v(b_c, q_c)$.

As $y \to 1$, this setting converges to our baseline model and the activist and conservative equilibria can co-exist. As $y \to 0$, the equilibrium becomes unique and converges to the no-trade benchmark.

The key difference that distinguishes the analysis with $y < 1$ from the baseline model is that the marginal voter can now be less extreme than the marginal trader if y is sufficiently close to zero. Intuitively, when y is very small, the supply of shares is very low, and only the most extreme shareholders, those with the highest willingness to pay, buy shares in equilibrium. In other words, the marginal trader is very extreme and, as $y \to 0$, converges to $-\bar{b}$ in the conservative equilibrium and to $\bar{b}$ in the activist equilibrium. By contrast, the post-trade shareholder base is very similar to the initial shareholder base because the volume of trade is low due to small y. As such, the marginal voter is relatively moderate and, as $y \to 0$, the marginal voter converges to $q_{NoTrade} \in \left(-\bar{b}, \bar{b}\right)$.

As in the baseline model, the expected welfare of the pre-trade shareholder base is equal to the expected welfare of the post-trade shareholder base, because prices are just transfers from buying to selling shareholders. However, different from the baseline model, since selling shareholders cannot exit their entire position in the firm, the expected welfare of the post-trade shareholder base is now a weighted average of the buying shareholders' expected welfare and the selling shareholders' expected welfare, where the weight on the former is always larger than the weight on the latter. To see this explicitly, consider, for example, a conservative equilibrium. Then, the expected shareholder welfare is

$$W_c = (1-y)\Pr[b > b_c]\,\mathbb{E}[v(b, q_c)\,|b > b_c] + (1+x)\Pr[b < b_c]\,\mathbb{E}[v(b, q_c)\,|b < b_c]. \tag{54}$$

Market clearing implies that $(1-y)\Pr[b > b_c] + (1+x)\Pr[b < b_c] = 1$, and hence, indeed, W_c is a weighted average of $\mathbb{E}[v(b, q_c)\,|b > b_c]$ and $\mathbb{E}[v(b, q_c)\,|b < b_c]$, the welfare of selling and buying shareholders, respectively. Since $(1-y)\Pr[b > b_c]$ decreases in y, the weight that is put on the selling shareholders is decreasing in y, as they hold a smaller and smaller fraction of the firm post-trade. As $y \to 0$, $W_c \to \mathbb{E}[v(b, q_{NoTrade})]$, and as $y \to 1$, $W_c \to \mathbb{E}[v(b, q_c)\,|b < b_c]$, just as in the baseline model.

As before, the expected shareholder welfare obtains its maximum exactly when the bias of the marginal voter is equal to the average bias of the post-trade shareholder. Thus, our results on the optimal majority rule and the optimal board naturally extend to this setup. For example, in the conservative equilibrium, the average bias of the post-trade shareholder base is $(1-y)\Pr[b > b_c]\,\mathbb{E}[b|b > b_c] + (1+x)\Pr[b < b_c]\,\mathbb{E}[b|b < b_c]$, which includes the biases of both buying and selling shareholders. Note that this bias is always strictly smaller than $\mathbb{E}[b]$, and similarly, the average bias of the post-trade shareholder base in the activist equilibrium is strictly larger than $\mathbb{E}[b]$. These observations imply that the bias of the optimal board is different from $\mathbb{E}[b]$ in this setup as well.

Finally, the extension to $y < 1$ allows us to revisit our results on the effect of liquidity on welfare when trading frictionshave a symmetric effect on buy and sell orders. For this purpose we impose $x = y$ and consider the effect of increasing x and y by the same amount, which can be interpreted as an increase in market depth. We show that the expected shareholder welfare decreases in market depth under similar conditions to those specified in Proposition 6 part (ii). For example, the expected welfare in the conservative equilibrium W_c decreases in market depth whenever $|1 - F(q_c) - \phi|$ is relatively small and the marginal voter in this equilibrium is more conservative than the average post-trade shareholder. The intuition is the same as in the baseline model.

A.4.2 Analysis

Define

$$\delta(y) \equiv \frac{x}{y+x}. \tag{55}$$

We prove the following results. The first result is the analog of Proposition 3.

Proposition 13. *Consider the setup of the baseline model where shareholders can only sell $y < 1$ of their shares. An equilibrium of the game with trading and voting always exists.*

*(i) A **conservative** equilibrium exists if and only if $q_c > F^{-1}(1-\phi)$, where*

$$q_c = \begin{cases} -G^{-1}\left((1-\delta(1))(1-\tau)\right) & \text{if } \delta(y)(1-y) \le \tau \\ -G^{-1}\left(\frac{1-\tau-y}{1-y}\right) & \text{if } \delta(y)(1-y) > \tau \end{cases}. \tag{56}$$

In this equilibrium, the shareholder buys x shares if $b < b_c$ and sells y shares if $b > b_c$, where

$$b_c = G^{-1}(1-\delta(y)). \tag{57}$$

The proposal is accepted if and only if $q > q_c$, and the share price is given by $p_c = v(b_c, q_c)$.

*(ii) An **activist** equilibrium exists if and only if $q_a < F^{-1}(1-\phi)$, where*

$$q_a = \begin{cases} -G^{-1}\left(1-(1-\delta(1))\tau\right) & \text{if } \delta(y)(1-y) \le 1-\tau \\ -G^{-1}\left(\frac{1-\tau}{1-y}\right) & \text{if } \delta(y)(1-y) > 1-\tau \end{cases}. \tag{58}$$

In this equilibrium, the shareholder buys x shares if $b > b_a$ and sells y shares if $b < b_a$, where

$$b_a \equiv G^{-1}(\delta(y)). \tag{59}$$

The proposal is accepted if and only if $q > q_a$, and the share price is given by $p_a = v(b_a, q_a)$.

(iii) Other equilibria do not exist.

The second result is the analog of Lemma 2.

Lemma 5. *In any equilibrium, the expected welfare of the shareholder base pre-trade is equal to the expected welfare of the shareholder base post-trade. In particular,*

$$W_c = (1-y)\Pr[b > b_c]\,\mathbb{E}[v(b,q_c)\,|b > b_c] + (1+x)\Pr[b < b_c]\,\mathbb{E}[v(b,q_c)\,|b < b_c] \tag{60}$$

and

$$W_a = (1-y)\Pr[b < b_a]\,\mathbb{E}[v(b,q_a)\,|b < b_a] + (1+x)\Pr[b > b_a]\,\mathbb{E}[v(b,q_a)\,|b > b_a]\,. \tag{61}$$

The third result is the analog of Lemma 3.

Lemma 6. *The expected welfare obtains its maximum exactly when the bias of the marginal voter is equal to the average bias of the post-trade shareholder, which is given by $(1-y)\,\delta(y)\,\mathbb{E}[b|b > b_c]+$ $(1+x)(1-\delta(y))\,\mathbb{E}[b|b < b_c]$ in the conservative equilibrium and by $(1-y)\,\delta(y)\,\mathbb{E}[b|b < b_a] +$ $(1+x)(1-\delta(y))\,\mathbb{E}[b|b > b_a]$ in the activist equilibrium. Moreover, the average bias of the post-trade shareholder in the conservative (activist) equilibrium is strictly smaller (larger) than $\mathbb{E}[b]$.*

Finally, the last result is the analog of Proposition 6 part (ii).

Proposition 14. *Suppose $x = y$. Then:*

(i) If a conservative equilibrium exists (i.e., $q_c > F^{-1}(1-\phi)$) then here exists $\phi_c > 1-F(q_c)$ such that the expected shareholder welfare in the conservative equilibrium W_c decreases in market depth (i.e., a change in y and in x by the same amount) if and only if $\phi \in (1-F(q_c), \phi_c)$ and the marginal voter in this equilibrium is more conservative than the average post-trade shareholder (i.e., $-q_c < (1-y)\,0.5\mathbb{E}[b|b > b_c]+(1+y)\,0.5\mathbb{E}[b|b < b_c]$).

(ii) If an activist equilibrium exists (i.e., $q_a < F^{-1}(1-\phi)$) then there exists $\phi_a < 1-F(q_a)$ such that the expected shareholder welfare in the activist equilibrium W_a decreases in market depth (i.e., a change in y and in x by the same amount) if and only if $\phi \in (\phi_a, 1-F(q_a))$ and the marginal voter in this equilibrium is less conservative than the average post-trade shareholder (i.e., $-q_a > (1-y)\,0.5\mathbb{E}[b|b < b_a] + (1+y)\,0.5\mathbb{E}[b|b > b_a]$).

A.4.3 Proofs

Proof of Proposition 13. Notice that Lemma 1 continues to hold in this setup and the expected value of shareholder b is given by (7). We consider three cases. First, suppose that $q^* > F^{-1}(1-\phi)$ (conservative equilibrium). The proof of Proposition 2 can be repeated in a setup with $y < 1$ to show that if $q^* > F^{-1}(1-\phi)$ then $v(b,q^*)$ decreases in b and therefore there exists b_c such that $v(b,q^*) > p \Leftrightarrow b < b_c$. The key difference is that the market clears if and only if

$$xG(b_c) = y(1-G(b_c)) \Leftrightarrow G(b_c) = 1-\delta(y)\,. \tag{62}$$

After the trading stage, shareholders with $b < b_c$, of which there $1-\delta(y)$, hold $1+x$ shares. Shareholders with $b > b_c$, of which there are $\delta(y)$, hold $1-y$ shares. Recall shareholder b

votes his shares for the proposal if and only if $q > -b$. Therefore, if $\delta(y)(1-y) \leq \tau$ then the marginal voter is among the buying shareholders, those with $b < b_c$, and if $\delta(y)(1-y) > \tau$ then the marginal voter is among the selling shareholders, those with $b > b_c$. Let us write the identity of the marginal voter explicitly. If $\delta(y)(1-y) \leq \tau$ then the marginal voter is determined by

$$\int_{-\bar{b}}^{-q_c} (1+x)\, dG(b) = 1-\tau \Leftrightarrow G(-q_c) = \frac{1-\tau}{1+x} \Leftrightarrow q_c = -G^{-1}((1-\delta(1))(1-\tau)),$$

just as in the baseline model (recall $\delta = \delta(1)$ in the baseline model). If $\delta(y)(1-y) > \tau$ then the marginal voter is determined by

$$\begin{aligned}
\int_{-\bar{b}}^{b_c} (1+x)\, dG(b) + \int_{b_c}^{-q_c} (1-y)\, dG(b) &= 1-\tau \Leftrightarrow \\
G(b_c)(1+x) + (1-y)(G(-q_c) - G(b_c)) &= 1-\tau \Leftrightarrow \\
G(b_c)(y+x) + (1-y)G(-q_c) &= 1-\tau \Leftrightarrow \\
\delta(y)(y+x) + (1-y)G(-q_c) &= 1-\tau \Leftrightarrow y + (1-y)G(-q_c) = 1-\tau \Leftrightarrow \\
q_c &= -G^{-1}\left(\frac{1-\tau-y}{1-y}\right),
\end{aligned}$$

as required. Hence, the cutoff in this "conservative" equilibrium is q_c as given by (56). Similarly to the proof of Proposition 2, the share price is $p_c = v(b_c, q_c)$.

Second, suppose that $q^* < F^{-1}(1-\phi)$ (activist equilibrium). The proof of Proposition 2 can be repeated in a setup with $y < 1$ to show that if $q^* < F^{-1}(1-\phi)$ then $v(b, q^*)$ increases in b and therefore there exists b_a such that $v(b, q^*) > p \Leftrightarrow b > b_c$. The key difference is that the market clears if and only if

$$x(1 - G(b_a)) = yG(b_a) \Leftrightarrow G(b_a) = \delta(y). \tag{63}$$

After the trading stage, shareholders with $b > b_a$, of which there $1-\delta(y)$, hold $1+x$ shares. Shareholders with $b < b_a$, of which there are $\delta(y)$, hold $1-y$ shares. Recall shareholder b votes his shares for the proposal if and only if $q > -b$. Therefore, if $\delta(y)(1-y) \leq 1-\tau$ then the marginal voter is among the buying shareholders, those with $b > b_a$, and if $\delta(y)(1-y) > 1-\tau$ then the marginal voter is among the selling shareholders, those with $b < b_a$. Let us write the identity of the marginal voter explicitly. If $\delta(y)(1-y) \leq 1-\tau$ then the marginal voter is determined by

$$\begin{aligned}
\int_{q_a}^{\bar{b}} (1+x)\, dG(b) &= \tau \Leftrightarrow \\
1 - G(q_a) &= \frac{\tau}{1+x} \Leftrightarrow \\
q_a &= -G^{-1}(1-(1-\delta(1))\tau),
\end{aligned}$$

just as in the baseline model. If $\delta(y)(1-y) > 1-\tau$ then the marginal voter is determined by

$$\begin{aligned}
\int_{q_a}^{b_a} (1-y)\, dG(b) + \int_{b_a}^{\bar{b}} (1+x)\, dG(b) = \tau \Leftrightarrow \\
(1-y)(G(b_a) - G(q_a)) + (1+x)(1-G(b_a)) = \tau \Leftrightarrow \\
G(q_a) = \frac{1+x-\tau-(y+x)G(b_a)}{1-y} \Leftrightarrow \\
G(q_a) = \frac{1-\tau}{1-y} q_a = -G^{-1}\left(\frac{1-\tau}{1-y}\right),
\end{aligned}$$

as required. Hence, the cutoff in this "conservative" equilibrium is q_a as given by (58). Similarly to the proof of Proposition 2, the share price is $p_a = v(b_a, q_a)$.

Third, the same arguments that are outlined in the proof of Proposition 2 to show that an equilibrium with $F(q^*) = 1-\phi$ does not exist, hold in this case as well.

Finally, notice that if $\delta(y)(1-y) \leq \min\{\tau, 1-\tau\}$ or $\delta(y)(1-y) \geq \max\{\tau, 1-\tau\}$ then $q_a < q_c$. Suppose $\tau < \delta(y)(1-y) < 1-\tau$ then $q_c = -G^{-1}(\frac{1-\tau-y}{1-y})$ and $q_a = -G^{-1}(1-(1-\delta(1))\tau)$, and $q_a < q_c$ if and only if $-y < x$ which always holds. If $1-\tau < \delta(y)(1-y) < \tau$ then $q_c = -G^{-1}((1-\delta(1))(1-\tau))$ and $q_a = -G^{-1}(\frac{1-\tau}{1-y})$, and $q_a < q_c$ if and only if $-y < x$ which always holds. Since $q_a < q_c$, in any event either $F(q_c) > 1-\phi$, $F(q_a) < 1-\phi$, or both. Therefore, an equilibrium always exists. ■

Proof of Lemma 5. The expected shareholder welfare in a conservative equilibrium is

$$\begin{aligned}
W_c &= \Pr[b > b_c]\,\mathbb{E}[(1-y)v(b,q_c) + yp_c | b > b_c] + \Pr[b < b_c]\,\mathbb{E}[(1+x)v(b,q_c) - xp_c | b < b_c] \\
&= \Pr[b > b_c]\, yp_c - \Pr[b < b_c]\, xp_c \\
&\quad + (1-y)\Pr[b > b_c]\,\mathbb{E}[v(b,q_c) | b > b_c] + (1+x)\Pr[b < b_c]\,\mathbb{E}[v(b,q_c) | b < b_c]
\end{aligned}$$

Notice that $\Pr[b < b_c] = \frac{y}{y+x}$ and hence

$$\Pr[b > b_c]\, yp_c - \Pr[b < b_c]\, xp_c = \left(\frac{x}{y+x} y - \frac{y}{y+x} x\right) p_c = 0$$

Then

$$W_c = (1-y)\Pr[b > b_c]\,\mathbb{E}[v(b,q_c) | b > b_c] + (1+x)\Pr[b < b_c]\,\mathbb{E}[v(b,q_c) | b < b_c],$$

as required. Similarly, the expected shareholder welfare in an activist equilibrium is

$$\begin{aligned}
W_a &= \Pr[b < b_a]\,\mathbb{E}[(1-y)v(b,q_a) + yp_a | b < b_a] + \Pr[b > b_a]\,\mathbb{E}[(1+x)v(b,q_a) - xp_a | b > b_a] \\
&= \Pr[b < b_a]\, yp_a - \Pr[b > b_a]\, xp_a \\
&\quad + (1-y)\Pr[b < b_a]\,\mathbb{E}[v(b,q_a) | b < b_a] + (1+x)\Pr[b > b_a]\,\mathbb{E}[v(b,q_a) | b > b_a]
\end{aligned}$$

Notice that $\Pr[b > b_a] = \frac{y}{y+x}$ and hence

$$\Pr[b < b_a]\, y p_a - \Pr[b > b_c]\, x p_a = 0$$

Then

$$W_a = (1-y)\Pr[b < b_a]\,\mathbb{E}[v(b,q_a)\,|b < b_a] + (1+x)\Pr[b > b_a]\,\mathbb{E}[v(b,q_a)\,|b > b_a],$$

as required. ■

Proof of Lemma 6. Notice that

$$\begin{aligned}\frac{\partial W_c}{\partial q^*} &= -f(q^*)\left[(1-y)\,\delta(y)\left(\mathbb{E}[b|b > b_c] + q^*\right) + (1+x)\left(1-\delta(y)\right)\left(\mathbb{E}[b|b < b_c] + q^*\right)\right] \\ &= -f(q^*)\left[(1-y)\,\delta(y)\,\mathbb{E}[b|b > b_c] + (1+x)\left(1-\delta(y)\right)\mathbb{E}[b|b < b_c] + q^*\right]\end{aligned}$$

Thus, the optimal cutoff satisfies

$$-q^* = (1-y)\,\delta(y)\,\mathbb{E}[b|b > b_c] + (1+x)\left(1-\delta(y)\right)\mathbb{E}[b|b < b_c].$$

Since $\delta(y)\,\mathbb{E}[b|b > b_c] + (1-\delta(y))\,\mathbb{E}[b|b < b_c] = \mathbb{E}[b]$ and $x(1-\delta(y)) = y\delta(y)$, we have

$$\begin{aligned}(1-y)\,\delta(y)\,\mathbb{E}[b|b > b_c] + (1+x)\left(1-\delta(y)\right)\mathbb{E}[b|b < b_c] &< \mathbb{E}[b] \Leftrightarrow \\ -y\delta(y)\,\mathbb{E}[b|b > b_c] + x\left(1-\delta(y)\right)\mathbb{E}[b|b < b_c] &< 0 \Leftrightarrow \\ x\left(1-\delta(y)\right)\mathbb{E}[b|b < b_c] &< y\delta(y)\,\mathbb{E}[b|b > b_c] \Leftrightarrow \\ \mathbb{E}[b|b < b_c] &< \mathbb{E}[b|b > b_c],\end{aligned}$$

which always holds.

Also notice that

$$\begin{aligned}\frac{\partial W_a}{\partial q^*} &= -f(q^*)\left[(1-y)\,\delta(y)\left(\mathbb{E}[b|b < b_a] + q^*\right) + (1+x)\left(1-\delta(y)\right)\left(\mathbb{E}[b|b > b_a] + q^*\right)\right] \\ &= -f(q^*)\left[(1-y)\,\delta(y)\,\mathbb{E}[b|b < b_a] + (1+x)\left(1-\delta(y)\right)\mathbb{E}[b|b > b_a] + q^*\right]\end{aligned}$$

Thus, the optimal cutoff satisfies

$$-q^* = (1-y)\,\delta(y)\,\mathbb{E}[b|b < b_a] + (1+x)\left(1-\delta(y)\right)\mathbb{E}[b|b > b_a].$$

Since $\delta(y)\,\mathbb{E}[b|b < b_a] + (1-\delta(y))\,\mathbb{E}[b|b > b_a] = \mathbb{E}[b]$ and $x(1-\delta(y)) = y\delta(y)$, we have

$$\begin{aligned}(1-y)\,\delta(y)\,\mathbb{E}[b|b < b_a] + (1+x)\left(1-\delta(y)\right)\mathbb{E}[b|b > b_a] &> \mathbb{E}[b] \Leftrightarrow \\ -y\delta(y)\,\mathbb{E}[b|b < b_a] + x\left(1-\delta(y)\right)\mathbb{E}[b|b > b_a] &> 0 \Leftrightarrow \\ x\left(1-\delta(y)\right)\mathbb{E}[b|b > b_a] &> y\delta(y)\,\mathbb{E}[b|b < b_a] \Leftrightarrow \\ \mathbb{E}[b|b > b_a] &> \mathbb{E}[b|b < b_a],\end{aligned}$$

as required. ■

Proof of Proposition 14. Notice that if $x = y$ then $\delta(y) = 0.5$, $b_c = b_a = G^{-1}(0.5)$, and from the expressions in Proposition 13 it can be verified that $\frac{\partial q_c(y)}{\partial y} > 0 > \frac{\partial q_a(y)}{\partial y}$.

Consider the conservative equilibrium first. In this case,

$$\begin{aligned} W_c &= (1-y)\,0.5\mathbb{E}\left[v(b,q_c)\,|b>b_c\right] + (1+y)\,0.5\mathbb{E}\left[v(b,q_c)\,|b<b_c\right] \\ &= (1-y)\,0.5\begin{pmatrix} \mathbb{E}[b|b>b_c](1-F(q_c)-\phi) \\ +v_0+(1-F(q_c))\,\mathbb{E}[\theta|q>q_c] \end{pmatrix} + (1+y)\,0.5\begin{pmatrix} \mathbb{E}[b|b<b_c](1-F(q_c)-\phi) \\ +v_0+(1-F(q_c))\,\mathbb{E}[\theta|q>q_c] \end{pmatrix} \end{aligned}$$

and

$$\begin{aligned} \frac{\partial W_c}{\partial y} &= -0.5\begin{pmatrix} \mathbb{E}[b|b>b_c](1-F(q_c)-\phi) \\ +v_0+(1-F(q_c))\,\mathbb{E}[\theta|q>q_c] \end{pmatrix} + 0.5\begin{pmatrix} \mathbb{E}[b|b<b_c](1-F(q_c)-\phi) \\ +v_0+(1-F(q_c))\,\mathbb{E}[\theta|q>q_c] \end{pmatrix} \\ &\quad -f(q_c)\frac{\partial q_c}{\partial y}\left[(1-y)\,0.5\mathbb{E}[b|b>b_c] + (1+y)\,0.5\mathbb{E}[b|b<b_c] + q_c\right] \\ &= -0.5\left(\mathbb{E}[b|b>b_c] - \mathbb{E}[b|b<b_c]\right)(1-F(q_c)-\phi) \\ &\quad -f(q_c)\frac{\partial q_c}{\partial y}\left[(1-y)\,0.5\mathbb{E}[b|b>b_c] + (1+y)\,0.5\mathbb{E}[b|b<b_c] + q_c\right] \end{aligned}$$

Therefore, $\frac{\partial \hat{W}_c^*}{\partial y} < 0$ if and only if

$$\phi < \phi_c \equiv (1-F(q_c)) + f(q_c)\frac{\partial q_c}{\partial y}\frac{(1-y)\,0.5\mathbb{E}[b|b>b_c] + (1+y)\,0.5\mathbb{E}[b|b<b_c] + q_c}{0.5\left(\mathbb{E}[b|b>b_c] - \mathbb{E}[b|b<b_c]\right)}$$

Recall the conservative equilibrium exists if and only if $\phi > 1 - F(q_c)$. Thus the interval in which $\frac{\partial \hat{W}_c^*}{\partial y} < 0$ is non-empty if and only if $1 - F(q_c) < \phi_c$, which holds if and only if $-q_c < (1-y)\,0.5\mathbb{E}[b|b>b_c] + (1+y)\,0.5\mathbb{E}[b|b<b_c]$. This completes part (i).

Consider the activist equilibrium. If $x = y$ then

$$\begin{aligned} W_a &= (1-y)\,0.5\mathbb{E}\left[v(b,q_a)\,|b<b_a\right] + (1+y)\,0.5\mathbb{E}\left[v(b,q_a)\,|b>b_a\right] \\ &= (1-y)\,0.5\begin{pmatrix} \mathbb{E}[b|b<b_a](1-F(q_a)-\phi) \\ +v_0+(1-F(q_a))\,\mathbb{E}[\theta|q>q_a] \end{pmatrix} + (1+y)\,0.5\begin{pmatrix} \mathbb{E}[b|b>b_a](1-F(q_a)-\phi) \\ +v_0+(1-F(q_a))\,\mathbb{E}[\theta|q>q_a] \end{pmatrix} \end{aligned}$$

and

$$\begin{aligned} \frac{\partial W_a}{\partial y} &= -0.5\begin{pmatrix} \mathbb{E}[b|b<b_a](1-F(q_a)-\phi) \\ +v_0+(1-F(q_a))\,\mathbb{E}[\theta|q>q_a] \end{pmatrix} + 0.5\begin{pmatrix} \mathbb{E}[b|b>b_a](1-F(q_a)-\phi) \\ +v_0+(1-F(q_a))\,\mathbb{E}[\theta|q>q_a] \end{pmatrix} \\ &\quad -f(q_a)\frac{\partial q_a}{\partial y}\left((1-y)\,0.5\mathbb{E}[b|b<b_a] + (1+y)\,0.5\mathbb{E}[b|b>b_a] + q_a\right) \\ &= 0.5\left(\mathbb{E}[b|b>b_a] - \mathbb{E}[b|b<b_a]\right)(1-F(q_a)-\phi) \\ &\quad -f(q_a)\frac{\partial q_a}{\partial y}\left((1-y)\,0.5\mathbb{E}[b|b<b_a] + (1+y)\,0.5\mathbb{E}[b|b>b_a] + q_a\right) \end{aligned}$$

Therefore, $\frac{\partial \hat{W}_a^*}{\partial y} < 0$ if and only if

$$\phi > \phi_a \equiv 1 - F(q_a) - f(q_a) \frac{\partial q_a}{\partial y} \frac{(1-y)\, 0.5 \mathbb{E}[b|b < b_a] + (1+y)\, 0.5 \mathbb{E}[b|b > b_a] + q_a}{0.5\left(\mathbb{E}[b|b > b_a] - \mathbb{E}[b|b < b_a]\right)}$$

Recall the activist equilibrium exists if and only if $\phi < 1 - F(q_a)$. Thus the interval in which $\frac{\partial \hat{W}_a^*}{\partial y} < 0$ is non-empty if and only if $1 - F(q_a) > \phi_a$, which holds if and only if $-q_a > (1-y)\, 0.5 \mathbb{E}[b|b < b_a] + (1+y)\, 0.5 \mathbb{E}[b|b > b_a]$. This completes part (ii).
■

B Supplementary results for the baseline model

B.1 Optimal majority requirement

Proposition 15. *The optimal majority requirement in the conservative equilibrium, denoted by τ_c, satisfies*

$$-q_c(\tau_c) = \min\{\beta_c, -H^{-1}(\phi)\}, \tag{64}$$

and the optimal majority requirement in the activist equilibrium, denoted by τ_a, satisfies

$$-q_a(\tau_a) = \max\{\beta_a, -H^{-1}(\phi)\} \tag{65}$$

where $q_c(\tau)$ and $q_a(\tau)$ are given by (11) and (10).[30]

Proof. Consider first the conservative equilibrium, which exists if and only if $H(q_c) < \phi$. Recall $W_c = v(\beta_c, q_c)$, where $b_c = G^{-1}(1-\delta)$, $\beta_c = \mathbb{E}[b|b < b_c]$, and $q_c = -G^{-1}((1-\delta)(1-\tau))$. Using (7),

$$\frac{\partial W_c}{\partial \tau} = -\left(\beta_c + q_c\right) \frac{\partial q_c}{\partial \tau} f(q_c)$$

Using (11), we get $\frac{\partial q_c}{\partial \tau} = \frac{1-\delta}{g(-q_c)} > 0$. Plugging into $\frac{\partial W_c}{\partial \tau}$, we get

$$\frac{\partial W_c}{\partial \tau} = -(1-\delta)\left(\beta_c + q_c\right) \frac{f(q_c)}{g(-q_c)},$$

and hence, $\frac{\partial W_c}{\partial \tau} > 0 \Leftrightarrow -q_c > \beta_c$. Recall that the conservative equilibrium exists if and only if $H(q_c) < \phi \Leftrightarrow -q_c < -H^{-1}(\phi)$. Also notice that $-q_c(\tau)$ spans $\left[-\bar{b}, b_c\right]$ as a decreasing function of τ, and $\beta_c \in \left(-\bar{b}, b_c\right)$. Therefore, there is a unique $\hat{\tau}_c \in (0,1)$ such that $-q_c(\hat{\tau}_c) = \beta_c$. Thus, if $\beta_c < -H^{-1}(\phi)$ then $\tau_c = \hat{\tau}_c$, and if $\beta_c \geq -H^{-1}(\phi)$ then the closet marginal voter to β_c that implies a conservative equilibrium is $-q^* = -H^{-1}(\phi)$. Thus, $-q_c(\tau_c) = \min\{\beta_c, -H^{-1}(\phi)\}$ as required.[31]

[30] We analyze the optimal threshold in a given equilibrium rather than across all equilibria, because when multiple equilibria exist, unless a selection is imposed, the optimal threshold is not well defined.

[31] As in the proof of Proposition 7, if $q^* = H^{-1}(\phi)$, the tie-breaking rule we adopt implies that no share-

Next, consider the activist equilibrium, which exists if and only if $H(q_a) > \phi$. Recall $W_a = v(\beta_a, q_a)$, where $b_a = G^{-1}(\delta)$, $\beta_a = \mathbb{E}[b|b > b_a]$, and $q_a = -G^{-1}(1 - \tau(1-\delta))$. Using (7),

$$\frac{\partial W_a}{\partial \tau} = -(\beta_a + q_a)\frac{\partial q_a}{\partial \tau} f(q_a).$$

Using (11), we get $\frac{\partial q_a}{\partial \tau} = \frac{1-\delta}{g(-q_a)} > 0$. Plugging into $\frac{\partial W_a}{\partial \tau}$, we get

$$\frac{\partial W_a}{\partial \tau} = -(1-\delta)(\beta_a + q_a)\frac{f(q_a)}{g(-q_a)},$$

and hence, $\frac{\partial W_a}{\partial \tau} > 0 \Leftrightarrow -q_a > \beta_a$. Recall that the activist equilibrium exists if and only if $H(q_a) > \phi \Leftrightarrow -q_a > -H^{-1}(\phi)$. Also notice that $-q_a(\tau)$ spans $\left[b_a, \overline{b}\right]$ as a decreasing function of τ, and $\beta_a \in \left(b_a, \overline{b}\right)$. Therefore, there is a unique $\hat{\tau}_a \in (0,1)$ such that $-q_a(\hat{\tau}_a) = \beta_a$. Thus, if $\beta_a > -H^{-1}(\phi)$ then $\tau_a = \hat{\tau}_a$, and if $\beta_a \leq -H^{-1}(\phi)$ then the closet marginal voter to β_a that implies an activist equilibrium is $-q^* = -H^{-1}(\phi)$. Thus, $-q_a(\tau_a) = \max\{\beta_a, -H^{-1}(\phi)\}$ as required. ■

B.2 Effect of δ on the benefits of delegation to the optimal board

In this section, we examine the effect of liquidity on the benefit from delegation. Specifically, we ask how the comparison between delegation to an optimal board and decision-making via shareholder voting depends on liquidity δ. For this purpose, we assume that whenever multiple equilibria exist in the voting game, shareholders will coordinate on the equilibrium with the highest expected welfare. Hence, we are interested in the benefit from delegation $D(\delta) \equiv W_m^* - max\{W_a, W_c\}$, where W_a and W_c are given by (16), and the expected welfare with the optimal board is given by

$$W_m^* = e \cdot \max\{v(\beta_c, -\beta_c), v(\beta_a, -\beta_a)\}. \tag{66}$$

From part (ii) of Proposition 7, $D(\delta) \geq 0$ for all $\delta \in (0,1)$. In addition, we prove the following result:

Lemma 7. *The expected welfare under the optimal board, W_m^*, is increasing in δ.*

A direct implication of this lemma is that if expected shareholder welfare in the voting equilibrium decreases with δ, which happens under the conditions identified in Proposition 6 part (ii), then the benefit from delegation to the optimal board increases in δ:

Corollary 4. *If the expected welfare in the voting equilibrium decreases in δ, then the benefit from delegation to the optimal board, $D(\delta)$, increases in δ.*

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holder trades. While this tie-breaking rule implies that the trading strategies of shareholders in the delegation equilibrium are not continuous in q^* when $q^* = H^{-1}(\phi)$, the expected welfare of shareholders is nevertheless continuous in q^* when $q^* = H^{-1}(\phi)$, which is the only relevant consideration in the derivation of the optimal majority requirement.

Next, we show that generally, the effect of liquidity on the benefit from delegation is ambiguous, and $D(\delta)$ may be increasing or decreasing. To see this, compare, for example, the conservative equilibrium in the voting game and delegation to an optimal conservative board. The welfare benefit of delegation is

$$D_c(\delta) \equiv v(\beta_c, -\beta_c) - W_c = \beta_c [F(q_c) - F(-\beta_c)] + \int_{-\beta_c} qdF(q) - \int_{q_c} qdF(q), \qquad (67)$$

and hence

$$\frac{\partial D_c}{\partial \delta} = \frac{\partial q_c}{\partial \delta}(q_c + \beta_c) f(q_c) + \frac{\partial \beta_c}{\partial \delta}(F(q_c) - F(-\beta_c)). \qquad (68)$$

Note that

$$q_c > -\beta_c \Leftrightarrow F(q_c) > F(-\beta_c).$$

Hence, the first and the second expression both change signs at $q_c = -\beta_c$. Since $\frac{\partial q_c}{\partial \delta} > 0$ and $\frac{\partial \beta_c}{\partial \delta} < 0$, the first expression is negative (positive) and the second expression is positive (negative) if $q_c + \beta_c < 0$ ($q_c + \beta_c > 0$). Therefore, $D_c(\delta)$ may be increasing or decreasing depending on the relative size of these expressions. Intuitively, if the marginal voter is more (less) conservative then the average post-trade shareholder (i.e., $-q_c < (>) \beta_c$), then an increase in liquidity, which makes the marginal voter in the voting game even more conservative ($\frac{\partial q_c}{\partial \delta} > 0$), increases (decreases) the benefit from delegation. On the other hand, an increase in liquidity also makes the average post-trade shareholder more conservative ($\frac{\partial \beta_c}{\partial \delta} < 0$), which increases the efficiency of the voting equilibrium and thus reduces (increases) the benefit from delegation.

The next result shows which of these effects dominates when liquidity is relatively high or low.

Proposition 16. *There exist* $0 < \underline{\delta} \leq \overline{\delta} < 1$ *such that:*

(*i*) *If* $G(\mathbb{E}[b]) \neq 1 - \tau$, *then* $\Delta(\delta') > \Delta(\delta'')$ *for all* $\delta' < \underline{\delta}$ *and* $\delta'' > \overline{\delta}$.

(*ii*) *If* $G(\mathbb{E}[b]) = 1 - \tau$, *then* $\Delta(\delta') > \Delta(\delta'')$ *for all* $\delta' \in (\underline{\delta}, \overline{\delta})$ *and* $\delta'' \notin (\underline{\delta}, \overline{\delta})$.

Intuitively, consider the generic case (i) in which the bias of the marginal voter in the no-trade equilibrium, $-q_{NoTrade}$, does not happen to be equal to the average bias of the pre-trade shareholder base. If liquidity is low and converges to zero ($\delta \to 0$), then both voting equilibria converge to the no-trade equilibrium, which is then strictly inferior to the case with optimal delegation in which marginal voter and the average shareholder are aligned. Conversely, as liquidity increases, extreme shareholders can build larger positions in the firm and tilt the voting outcome in their favor more often, which reduces the benefit of delegation to an optimal board. Indeed, in the limit, as liquidity becomes large ($\delta \to 1$), both the marginal trader and the marginal voter converge to the most extreme shareholder, so their preferences are fully aligned and delegation adds no value. Hence, the benefits of delegation are large if liquidity is low and small if liquidity is large. The case in which the no-trade equilibrium is efficient is different, because then the voting equilibria converge to an efficient no-trade equilibrium, delegation adds no value and the benefits from delegation arise only for intermediate values of liquidity (part (ii) of the proposition).

Proof of Lemma 7. It follows from (66) that $W_m^* = e \cdot v(B, -B)$, where B is either β_c or β_a. We first prove that $v(B, -B)$ increases in B if and only if $H(-B) > \phi$. Indeed, based on (7),

$$\begin{aligned} v(B,-B) &= v_0 + B(H(-B) - \phi) + H(-B)\mathbb{E}[\theta | q > -B] \\ &= v_0 - \phi B + \int_{-B} (q + B)\, dF(q) \end{aligned} \tag{69}$$

if $-B \in (-\Delta, \Delta)$. If $-B > \Delta$, the last term in (69) is zero, and if $-B < -\Delta$, the last term is B. Therefore, $\frac{\partial v(B,-B)}{\partial B} = H(-B) - \phi$ for all $-B$, as required.

From (66), W_m^* depends on δ only through its effect on b_a and b_c. First, suppose $\phi > \Phi$. Then $b_m^* = \beta_c$, and the equilibrium under the optimal board is conservative in the sense that $H(-\beta_c) < \phi$. Then, $W_m^* = e \cdot v(B, -B)|_{B=\beta_c}$ and $\frac{\partial v(B,-B)}{\partial B}|_{B=\beta_c} < 0$. Since β_c decreases in δ, it follows that W_m^* increases in δ. Second, suppose $\phi < \Phi$. Then $b_m^* = \beta_a$, and the equilibrium under the optimal board is activist in the sense that $H(-\beta_a) > \phi$. Then, $W_m^* = e \cdot v(B, -B)|_{B=\beta_a}$ and $\frac{\partial v(B,-B)}{\partial B}|_{B=\beta_a} > 0$. Since β_a increases in δ, it follows that W_m^* increases in δ. Thus, if $\phi \neq \Phi$, then W_m^* increases in δ. If $\phi = \Phi$, then (25) implies $W_m^* = e \cdot v(\beta_c, -\beta_c) = e \cdot v(\beta_a, -\beta_a)$, and since both terms increase in δ, so does W_m^*. ■

Proof of Proposition 16. Recall that $\lim_{\delta \to 1} b_c = -\bar{b}$ and $\lim_{\delta \to 1} b_a = \bar{b}$. Then we have $\lim_{\delta \to 1} \beta_c = -\bar{b}$ and $\lim_{\delta \to 1} \beta_a = \bar{b}$. Also recall that $\lim_{\delta \to 1}(-q_c) = -\bar{b}$ and $\lim_{\delta \to 1}(-q_a) = \bar{b}$. Therefore, in both the voting game and the delegation game, the marginal trader and the decision-maker (marginal voter or board, respectively) converge to the most extreme shareholder. Therefore, the expected welfare of shareholders in both cases is the same and equals the valuation of the most extreme shareholder. This means that $\lim_{\delta \to 1} D(\delta) = 0$.

Next, consider the limit $\delta \to 0$. Recall that $\lim_{\delta \to 0} q_c = \lim_{\delta \to 0} q_a = q_{NoTrade}$ and $\lim_{\delta \to 0} b_c = \bar{b}$ and $\lim_{\delta \to 0} b_a = -\bar{b}$, and hence, $\lim_{\delta \to 0} \beta_c = \lim_{\delta \to 0} \beta_a = \mathbb{E}[b]$.

First, consider the case where $-q_{NoTrade} \neq \mathbb{E}[b]$. Since $v(\mathbb{E}[b], q)$ achieves its maximum at $q = -\mathbb{E}[b]$, we have

$$\lim_{\delta \to 0} W_m^* = e \cdot v(\mathbb{E}[b], -\mathbb{E}[b]) > e \cdot v(\mathbb{E}[b], q_{NoTrade}) = W_{NoTrade}^* = \lim_{\delta \to 0} W_v^*.$$

Thus, in this case, $\lim_{\delta \to 0} D(\delta) > 0$. Combining it with $\lim_{\delta \to 1} D(\delta) = 0$ and using the continuity of W_m^* and W_v^* in δ, implies that $D(\delta') > D(\delta'')$ for all δ' sufficiently close to 0 and all δ'' sufficiently close to 1, which proves the statement in part (i).

Second, consider the case where $-q_{NoTrade} = \mathbb{E}[b]$. Then $\lim_{\delta \to 0} W_m^* = \lim_{\delta \to 0} W_v^*$, so $\lim_{\delta \to 0} D(\delta) = 0$. Since $\lim_{\delta \to 1} D(\delta) = 0$, to prove the statement in part (ii), it is sufficient to show that there exists $\delta \in (0, 1)$ such that the benefit of delegation to an optimal board is strictly positive, i.e., the bias of the marginal voter in the voting game is different form the post-trade average shareholder. This follows immediately from the fact that q_c and q_a are equal to β_c or β_a only under knife-edge conditions on parameters. ■

B.3 Supplementary analysis for the proofs of the baseline results

In this section, we include the proofs of several results in the baseline model for cases in which the voting equilibrium is expected to be conservative.

Supplementary analysis for the proof of Lemma 2. Suppose that the voting equilibrium is conservative. Then, market clearing implies $\Pr[b > b_c]\, e = \Pr[b < b_c]\, x$, where $\Pr[b < b_c] = 1 - \delta = \frac{e}{x+e}$. Therefore,

$$\begin{aligned} W_c &= \Pr[b > b_c]\, e p_c + \Pr[b < b_c]\, \mathbb{E}[(e+x)\, v(b, q_c) - x p_c | b < b_c] \\ &= \Pr[b < b_c]\, x p_c + \Pr[b < b_c]\, \mathbb{E}[(e+x)\, v(b, q_c) - x p_c | b < b_c] \\ &= \Pr[b < b_c]\, \mathbb{E}[(e+x)\, v(b, q_c) | b < b_c] = (1-\delta)(e+x)\, \mathbb{E}[v(b, q_c) | b < b_c] \\ &= e\mathbb{E}[v(b, q_c) | b < b_c] = ev(\mathbb{E}[b | b < b_c], q_c) = ev(\beta_c, q_c). \end{aligned}$$

■

Supplementary analysis for the proof of Proposition 5. Consider the conservative equilibrium. Recall that in this equilibrium $W_c = e \cdot v(\beta_c, q_c)$ and $p_c = v(b_c, q_c)$. Then, a change in parameters that affects the marginal voter (q_c) without changing the marginal trader only affects W_c and p_c through its effect on q_c. Also recall that based on (17), $v(\beta_c, q^*)$ is a hump-shaped function in q^* with a maximum at $q^* = -\beta_c$, and $v(b_c, q^*)$ is a hump-shaped function in q^* with a maximum at $q^* = -b_c$. Since $-b_c < q_c - \beta_c$ by assumption of the proposition, any small enough change in parameters that leaves this order unchanged ($-b_c < q_c - \beta_c$) either increases the distance of q_c to $-\beta_c$ but decreases the distance to $-b_c$, or vice versa. Hence, this change of parameters necessarily moves prices and welfare in opposite directions. ■

Supplementary analysis for the proof of Proposition 6. Consider the conservative equilibrium, which exists if and only if $H(q_c) < \phi$. Recall $p_c = v(b_c, q_c)$ and $W_c = e \cdot v(\beta_c, q_c)$, where $b_c = G^{-1}(1-\delta)$, $\beta_c = \mathbb{E}[b | b < b_c] = \frac{1}{G(b_c)} \int_{-\bar{b}}^{b_c} b dG(b)$, and $q_c = -G^{-1}((1-\delta)(1-\tau))$. Using (7),

$$\frac{\partial p_c}{\partial \delta} = \frac{\partial b_c}{\partial \delta}(H(q_c) - \phi) - (b_c + q_c)\frac{\partial q_c}{\partial \delta} f(q_c) \tag{70}$$

and

$$\frac{1}{e}\frac{\partial W_c}{\partial \delta} = \frac{\partial \beta_c}{\partial \delta}(H(q_c) - \phi) - (\beta_c + q_c)\frac{\partial q_c}{\partial \delta} f(q_c). \tag{71}$$

More precisely, (70)-(71) hold when $q_c \in (-\Delta, \Delta)$, and when q_c is outside these bounds, the second term in both of these expressions is equal to zero (as noted above, we set $f(q^*) = 0$ for $q^* \notin (-\Delta, \Delta)$).

Using (11) and (9), we get $\frac{\partial q_c}{\partial \delta} = \frac{1-\tau}{g(-q_c)} > 0$, $\frac{\partial b_c}{\partial \delta} = -\frac{1}{g(b_c)} < 0$, and

$$\frac{\partial \beta_c}{\partial \delta} = \frac{\frac{\partial b_c}{\partial \delta} b_c g(b_c) G(b_c) - \left[\int_{-\bar{b}}^{b_c} b g(b)\, db\right] g(b_c) \frac{\partial b_c}{\partial \delta}}{\left[G(b_c)\right]^2} = \frac{\partial b_c}{\partial \delta} \frac{g(b_c)}{G(b_c)} (b_c - \beta_c) = -\frac{b_c - \beta_c}{G(b_c)} < 0.$$

Plugging into (70) and (71), we get

$$\begin{aligned} \frac{\partial p_c}{\partial \delta} &= -\frac{H(q_c) - \phi}{g(b_c)} - (1-\tau)(b_c + q_c) \frac{f(q_c)}{g(-q_c)} \\ \frac{1}{e} \frac{\partial W_c}{\partial \delta} &= -\frac{H(q_c) - \phi}{G(b_c)} (b_c - \beta_c) - (1-\tau)(\beta_c + q_c) \frac{f(q_c)}{g(-q_c)}, \end{aligned}$$

where again, the second term is zero if $q_c \notin (-\Delta, \Delta)$. Notice that as $\delta \to 1$, then b_c, β_c, and $-q_c$ all converge to $-\bar{b}$, and $H(q_c) - \phi \to H(\bar{b}) - \phi$. Suppose the conservative equilibrium exists in the limit (which is the case if $H(\bar{b}) < \phi$). Since g is positive on $[-\bar{b}, \bar{b}]$,

$$\lim_{\delta \to 1} \frac{\partial p_c}{\partial \delta} = -\frac{H(\bar{b}) - \phi}{g(-\bar{b})} > 0.$$

In addition, $\lim_{\delta \to 1} \frac{1}{e} \frac{\partial W_c}{\partial \delta} = -\left(H(\bar{b}) - \phi\right) \lim_{\delta \to 1} \frac{b_c - \beta_c}{G(b_c)}$. Using l'Hopital's rule,

$$\lim_{\delta \to 1} \frac{b_c - \beta_c}{G(b_c)} = \lim_{\delta \to 1} \frac{\frac{\partial b_c}{\partial \delta} - \frac{\partial \beta_c}{\partial \delta}}{g(b_c) \frac{\partial b_c}{\partial \delta}} = \frac{1}{g(-\bar{b})} - \lim_{\delta \to 1} \frac{b_c - \beta_c}{G(b_c)},$$

which implies $\lim_{\delta \to 1} \frac{b_c - \beta_c}{G(b_c)} = \frac{1}{2} \frac{1}{g(-\bar{b})} > 0$, and hence $\lim_{\delta \to 1} \frac{\partial W_c}{\partial \delta} > 0$.

Also notice that as $\delta \to 0$, then $b_c \to \bar{b}$, $\beta_c \to \mathbb{E}[b]$, and $q_c \to q_{NoTrade} = -G^{-1}(1-\tau) > -\bar{b}$. Suppose the conservative equilibrium exists in this limit (which is the case if $H(q_{NoTrade}) < \phi$). Then, using (70),

$$\lim_{\delta \to 0} \frac{\partial p_c}{\partial \delta} = -\frac{H(q_{NoTrade}) - \phi}{g(\bar{b})} - (1-\tau)\left(\bar{b} + q_{NoTrade}\right) \frac{f(q_{NoTrade})}{g(-q_{NoTrade})},$$

where the second term is strictly negative because (1) by assumption, $q_{NoTrade} \in (-\Delta, \Delta)$, and (2) $\bar{b} + q_{NoTrade} > 0$, as shown above. Hence, $\lim_{\delta \to 0} \frac{\partial p_c}{\partial \delta} < 0$ if $|H(q_{NoTrade}) - \phi|$ is sufficiently small.

Also notice that

$$\lim_{\delta \to 0} \frac{1}{e} \frac{\partial W_c}{\partial \delta} = -\frac{H(q_{NoTrade}) - \phi}{G(\bar{b})} \left(\bar{b} - \mathbb{E}[b]\right) - (1-\tau)\left(\mathbb{E}[b] + q_{NoTrade}\right) \frac{f(q_{NoTrade})}{g(-q_{NoTrade})}.$$

Thus, if $\lim_{\delta \to 0} (\beta_c + q_c) = \mathbb{E}[b] + q_{NoTrade} > 0$ (i.e., the marginal voter is more extreme than the average post-trade shareholder) and $|H(q_{NoTrade}) - \phi|$ is small enough, then $\lim_{\delta \to 0} \frac{\partial W_c}{\partial \delta} < 0$. ■

Supplementary analysis for the proof of Proposition 8. Suppose the voting equilibrium is expected to be conservative. In this case, the expected payoff of shareholder b is

$$V_c(b, q^*) = \begin{cases} (e+x)\, v(b, q^*) - xv(b_c, q^*) & \text{if } b < b_c \\ ev(b_c, q^*) & \text{if } b \geq b_c. \end{cases} \tag{72}$$

Similarly, if shareholder b expects the delegation (to a board with bias $b_m = -q_m$) equilibrium to be conservative, his expected payoff is $V_c(b, q_m)$. Recall that the delegation equilibrium is conservative if and only if $H(q_m) < \phi \Leftrightarrow -q_m < -H^{-1}(\phi)$.

Consider as an alternative a conservative board with bias $b_m = -q_m < -H^{-1}(\phi)$. Shareholder b prefers delegation to such a board over the conservative voting equilibrium if and only if $V_c(b, q_c) < V_c(b, q_m)$. We consider several cases:

1. If $b \geq b_c$, then

$$\begin{aligned} V_c(b, q_c) &< V_c(b, q_m) \Leftrightarrow v(b_c, q_c) < v(b_c, q_m) \Leftrightarrow \\ b_c(H(q_c) - H(q_m)) &< H(q_m)\, \mathbb{E}[\theta | q > q_m] - H(q_c)\, \mathbb{E}[\theta | q > q_c]. \end{aligned}$$

- If in addition $q_c < q_m$, then $H(q_c) - H(q_m) > 0$, so

$$V_c(b, q_c) < V_c(b, q_m) \Leftrightarrow b_c < \mathbb{E}[-q| - q_m < -q < -q_c],$$

  which never holds since $b_c > -q_c$. Thus, shareholders with $b \geq b_c$ never support delegation to a board who is more extreme than the marginal voter, i.e., $q_c < q_m \Leftrightarrow b_m < -q_c$.

- If instead $q_c > q_m$, then $H(q_c) - H(q_m) < 0$, so

$$V_c(b, q_c) < V_c(b, q_m) \Leftrightarrow b_c > \mathbb{E}[-q| - q_c < -q < -q_m].$$

  Since $b_c > -q_c$, this always holds if $b_c \geq -q_m$ and might even hold if $b_c < -q_m$. Thus, shareholders with $b \geq b_c$ support delegation to a board whenever $-q_m \in (-q_c, b_c]$, and might even do so if $-q_m > b_c$.

2. If $b < b_c$, then (2) and (72) imply

$$\begin{gathered} V_c(b, q_c) < V_c(b, q_m) \Leftrightarrow v(b, q_c) - \delta v(b_c, q_c) < v(b, q_m) - \delta v(b_c, q_m) \Leftrightarrow \\ v(b, q_c) - v(b, q_m) < \delta [v(b_c, q_c) - v(b_c, q_m)] \Leftrightarrow \\ b(H(q_c) - H(q_m)) + H(q_c)\, \mathbb{E}[\theta | q > q_c] - H(q_m)\, \mathbb{E}[\theta | q > q_m] \\ < \delta [b_c(H(q_c) - H(q_m)) + H(q_c)\, \mathbb{E}[\theta | q > q_c] - H(q_m)\, \mathbb{E}[\theta | q > q_m]\ ]. \end{gathered}$$

- If in addition $q_c < q_m$, then

$$V_c(b, q_c) < V_c(b, q_m) \Leftrightarrow b < \delta b_c + (1 - \delta)\, \mathbb{E}[-q| - q_m < -q < -q_c],$$

  and notice that since $-q_c < b_c$, then $\delta b_c + (1 - \delta)\, \mathbb{E}[-q| - q_m < -q < -q_c] < b_c$.

- If instead $q_c > q_m$, then

$$V_c(b, q_c) < V_c(b, q_m) \Leftrightarrow b > \delta b_c + (1-\delta)\mathbb{E}[-q| - q_c < -q < -q_m].$$

The overall support for delegation to the board is the combined support of shareholders with $b < b_c$ and $b > b_c$. Then:
(i) First, consider a board with $-q_m < -q_c \Leftrightarrow q_m > q_c$. Then only shareholders with $b < \delta b_c + (1-\delta)\mathbb{E}[-q| - q_m < -q < -q_c] < b_c$ support delegation to the board. It follows that if $G(b_c) < \tau \Leftrightarrow 1 - \delta < \tau$, then this type of board does not obtain τ-support.
(ii) Second, consider a board with $-q_m > -q_c \Leftrightarrow q_m < q_c$. Such a board obtains support from $b \geq b_c$ if $b_c > \mathbb{E}[-q| - q_c < -q < -q_m]$ and from $b < b_c$ that satisfy $b > \delta b_c + (1-\delta)\mathbb{E}[-q| - q_c < -q < -q_m]$. There are two cases:

- If $b_c < \mathbb{E}[-q| - q_c < -q < -q_m]$, then $\delta b_c + (1-\delta)\mathbb{E}[-q| - q_c < -q < -q_m] > b_c$. Thus, in this case, there is no support for delegation from either shareholders with $b \geq b_c$ or from those with $b < b_c$.

- If $b_c > \mathbb{E}[-q| - q_c < -q < -q_m]$, then $\delta b_c + (1-\delta)\mathbb{E}[-q| - q_c < -q < -q_m] < b_c$. Thus, both shareholders with $b \geq b_c$ and with $b \in (\delta b_c + (1-\delta)\mathbb{E}[-q| - q_c < -q < -q_m], b_c)$ support delegation. So overall, delegation receives support from shareholders with $b > \delta b_c + (1-\delta)\mathbb{E}[-q| - q_c < -q < -q_m]$. Notice that $\mathbb{E}[-q| - q_c < -q < -q_m] > -q_c$, and hence the fraction of initial shareholders supporting delegation is

$$1 - G(\delta b_c + (1-\delta)\mathbb{E}[-q| - q_c < -q < -q_m]) < 1 - G(\delta b_c - (1-\delta) q_c).$$

  Since $\lim_{\tau \to 1} q_c = \bar{b}$, we have $\lim_{\tau \to 1} 1 - G(\delta b_c - (1-\delta) q_c) = 1 - G\left(\delta b_c - (1-\delta)\bar{b}\right) < 1$.

Combining (i) and (ii), we conclude that as $\tau \to 1$, no conservative board gains τ-support from shareholders if they expect the conservative voting equilibrium. ■