Online Appendix for
"Creating Controversy in Proxy Voting Advice"

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The Online Appendix is organized as follows. Section A contains the additional proofs for the main results (Lemma 1, the second part of Proposition 4, Proposition 8, the extensions presented in Section 4), as well as a more in-depth derivation of the value from buying the report. Section B contains the additional derivations for the empirical implications that are presented in Section 5 of the paper.

A Additional proofs for the main results

A.1 Proof of Lemma 1

Plugging (4) from Proposition 2 into (22) and simplifying the expression,

$$\Pr\left(Piv|q,\mu_s\right) = 2C_{N-1}^{\frac{N-1}{2}} q^{\frac{N-1}{2}} \left(\frac{\sqrt{\left(z\left(\mu_s\right)-1\right)^2+4q^2 z\left(\mu_s\right)}-q\left(1+z\left(\mu_s\right)\right)}{\left(z\left(\mu_s\right)-1\right)^2}\right)^{\frac{N-1}{2}} \mu_s,$$

where $z\left(\mu_s\right) \equiv \left(\frac{\mu_s}{1-\mu_s}\right)^{\frac{2}{N-1}}$. Define $\Omega\left(\mu_s|q\right)$ as

$$\Omega\left(\mu_s|q\right) \equiv \left(\frac{\varphi\left(z\left(\mu_s\right)\right)}{\left(z\left(\mu_s\right)-1\right)^2}\right)^{\frac{N-1}{2}} \mu_s, \tag{38}$$

where

$$\varphi\left(z\right) \equiv \sqrt{\left(z-1\right)^2+4q^2 z}-q\left(1+z\right). \tag{39}$$

Then, $\Pr\left(Piv|q,\mu_s\right) = 2C_{N-1}^{\frac{N-1}{2}} q^{\frac{N-1}{2}} \Omega\left(\mu_s|q\right)$, so $\Pr\left(Piv|q,\mu_s\right)$ is increasing (decreasing) and concave (convex) in μ_s at some $\left(\mu_s,q\right)$ if and only if $\Omega\left(\mu_s|q\right)$ is increasing (decreasing) and concave (convex) in μ_s at this $\left(\mu_s,q\right)$. Taking the first and second derivative of $z\left(\mu_s\right)$ and the first four derivatives of $\varphi(z)$, and dropping the subscript s in μ_s for ease of notation, we get:

$$
\begin{aligned}
z'(\mu) &= \frac{2}{N-1}\frac{z(\mu)}{\mu(1-\mu)}, && (40)\\
z''(\mu) &= \frac{2}{N-1}\frac{\frac{2}{N-1}-(1-2\mu)}{\mu^2(1-\mu)^2}z(\mu), && (41)\\
\varphi'(z) &= \frac{z-1+2q^2}{\sqrt{(z-1)^2+4q^2z}}-q, && (42)\\
\varphi''(z) &= \frac{4q^2(1-q^2)}{\left((z-1)^2+4q^2z\right)^{\frac{3}{2}}} && (43)\\
\varphi'''(z) &= \frac{-12q^2(1-q^2)(z-1+2q^2)}{\left((z-1)^2+4q^2z\right)^{\frac{5}{2}}} && (44)\\
\varphi''''(z) &= \frac{60q^2(1-q^2)(z-1+2q^2)^2}{\left((z-1)^2+4q^2z\right)^{\frac{7}{2}}}-\frac{12q^2(1-q^2)}{\left((z-1)^2+4q^2z\right)^{\frac{5}{2}}} && (45)
\end{aligned}
$$

We first prove the convexity/concavity properties of $\Omega(\mu|q)$. Differentiating $\Omega(\mu|q)$ twice with respect to μ and using z to denote $z(\mu)$ for ease of notation, we get

$$
\begin{aligned}
\Omega''(\mu|q) &= \Omega'(\mu|q)\left(\frac{N-1}{2}\left(\frac{\varphi'(z)}{\varphi(z)}-\frac{2}{z-1}\right)z'(\mu)+\frac{1}{\mu}\right) && (46)\\
&\quad+\Omega(\mu|q)\left(\begin{array}{c}\frac{N-1}{2}\left(\left[\frac{\varphi'(z)}{\varphi(z)}\right]'+\frac{2}{(z-1)^2}\right)(z'(\mu))^2\\ +\frac{N-1}{2}\left(\frac{\varphi'(z)}{\varphi(z)}-\frac{2}{z-1}\right)z''(\mu)-\frac{1}{\mu^2}\end{array}\right).
\end{aligned}
$$

Since $\mu\in(0,1)$, $q\in(0,1)$, and $\pi(q,\mu)\in(0,1)$, we have $\Pr(Piv|q,\mu)>0$ and hence $\Omega(\mu|q)>0$. Then, using (40)-(41) and simplifying,

$$
\begin{aligned}
\frac{\Omega''(\mu|q)}{\Omega(\mu|q)}\mu^{\frac{2N-4}{N-1}} &= \left(\begin{array}{c}\left(\frac{\varphi'(z)}{\varphi(z)}-\frac{2}{z-1}\right)^2+\\ +\frac{2}{N-1}\left(\frac{\varphi''(z)\varphi(z)-(\varphi'(z))^2}{\varphi(z)^2}+\frac{2}{(z-1)^2}\right)\end{array}\right)\mu^{\frac{2}{N-1}}(1-\mu)^{-\frac{2(N+1)}{N-1}}\\
&+2\left(\frac{\varphi'(z)}{\varphi(z)}-\frac{2}{z-1}\right)(1-\mu)^{-\frac{N+1}{N-1}}+\left(\frac{\varphi'(z)}{\varphi(z)}-\frac{2}{z-1}\right)\left(\frac{2}{N-1}-(1-2\mu)\right)(1-\mu)^{-\frac{2}{N}}.
\end{aligned}
$$

1) First, consider the limit case of $\mu\to 0$. When $\mu\to 0$, $\lim_{\mu\to 0}z(\mu)=0$, $\lim_{\mu\to 0}\varphi(z)=1-q$, $\lim_{\mu\to 0}\varphi'(z)=(q-1)(2q+1)$, and $\lim_{\mu\to 0}\varphi''(z)=4q^2(1-q^2)$. Therefore,

$$
\lim_{\mu\to 0}\frac{\Omega''(\mu|q)\mu^{\frac{2N-4}{N-1}}}{\Omega(\mu|q)}=\frac{N+1}{N-1}(1-2q).
$$

Since $\Omega(\mu|q) > 0$, we have $\lim_{\mu\to 0}\Omega''(\mu|q) > 0$ if and only if $q < \frac{1}{2}$. By continuity of the second derivative, there exists $\underline{\mu}$ such that: 1) if $q < \frac{1}{2}$, then $\Omega''(\mu|q) > 0$ for $\mu \in (0, \underline{\mu})$ and 2) if $q > \frac{1}{2}$, then $\Omega''(\mu|q) < 0$ for $\mu \in (0, \underline{\mu})$.

2) Second, consider the limit case of $\mu \to 1$. By symmetry of $\Omega(\mu|q)$ around $\mu = \frac{1}{2}$, this case is identical to $\mu \to 0$, so there exists $\bar{\mu}$ such that: 1) if $q < \frac{1}{2}$, then $\Omega''(\mu|q) > 0$ for $\mu \in (\bar{\mu}, 1)$ and 2) if $q > \frac{1}{2}$, then $\Omega''(\mu|q) < 0$ for $\mu \in (\bar{\mu}, 1)$.

3) Third, consider the limit case of $\mu \to \frac{1}{2}$. Using $\Omega'\left(\frac{1}{2}|q\right) = 0$ and the expressions (40)-(41), (46) at $\mu = \frac{1}{2}$ yields

$$\frac{\Omega''\left(\frac{1}{2}|q\right)}{\Omega\left(\frac{1}{2}|q\right)} = \frac{32}{N-1}\lim_{z\to 1}\left(\frac{\varphi''(z)\varphi(z) - (\varphi'(z))^2}{\varphi(z)^2} + \frac{2}{(z-1)^2}\right) + \frac{32}{N-1}\lim_{z\to 1}\left(\frac{\varphi'(z)}{\varphi(z)} - \frac{2}{z-1}\right) - 4.$$

Consider the second limit. Notice that $\varphi(1) = \varphi'(1) = 0$ and $\varphi''(1) \neq 0$. Applying l'Hopital's rule three times,

$$\begin{aligned}\lim_{z\to 1}\left(\frac{\varphi'(z)}{\varphi(z)} - \frac{2}{z-1}\right) &= \lim_{z\to 1}\left(\frac{\varphi'(z)(z-1) - 2\varphi(z)}{\varphi(z)(z-1)}\right)\\ &= \lim_{z\to 1}\left(\frac{\varphi''''(z)(z-1) + \varphi'''(z)}{\varphi'''(z)(z-1) + 3\varphi''(z)}\right) = \frac{\varphi'''(1)}{3\varphi''(1)} = -\frac{1}{2},\end{aligned}$$

where the last transition is from evaluating (43) and (44) at $z = 1$. Consider the first limit. Using $\lim_{z\to 1}\left(\frac{\varphi'(z)}{\varphi(z)} - \frac{2}{z-1}\right)^2 = \frac{1}{4}$ and applying l'Hopital's rule four times,

$$\begin{aligned}\lim_{z\to 1}\left(\frac{\varphi''(z)\varphi(z) - (\varphi'(z))^2}{\varphi(z)^2} + \frac{2}{(z-1)^2}\right) &= \lim_{z\to 1}\left(\frac{\varphi''(z)(z-1)^2 + 6\varphi(z) - 4\varphi'(z)(z-1)}{\varphi(z)(z-1)^2}\right) - \frac{1}{4}\\ &= \lim_{z\to 1}\left(\frac{\varphi''''''(z)(z-1)^2 + 4(z-1)\varphi'''''(z) + 2\varphi''''(z)}{\varphi''''(z)(z-1)^2 + 8(z-1)\varphi'''(z) + 12\varphi''(z)}\right) - \frac{1}{4}\\ &= \frac{\varphi''''(1)}{6\varphi''(1)} - \frac{1}{4} = \frac{\frac{3\left(1-q^2\right)\left(5q^2-1\right)}{8q^3}}{6\frac{1-q^2}{2q}} - \frac{1}{4} = \frac{3q^2-1}{8q^2},\end{aligned}$$

where the transition on the last line is from evaluating (43) and (45) at $z = 1$. Hence,

$$\lim_{z\to 1}\frac{\Omega''\left(\frac{1}{2}|q\right)}{\Omega\left(\frac{1}{2}|q\right)} = \frac{32}{N-1}\frac{3q^2-1}{8q^2} - \frac{32}{N-1}\frac{1}{2} - 4 = \frac{4}{N-1}\left(-1 - \frac{1}{q^2}\right) - 4 < 0.$$

Since $\Omega\left(\frac{1}{2}|q\right) > 0$, then for any q, $\Pr(Piv|q,\mu)$ is strictly concave at $\mu = \frac{1}{2}$ and, by continuity of the second derivative, there exists $\varepsilon > 0$ such that it is also strictly concave in $\mu \in \left(\frac{1}{2} - \varepsilon, \frac{1}{2} + \varepsilon\right)$.

A.2 Additional analysis for the proof of Proposition 4

The main appendix contains the proof of part 1 of the proposition, which applies to small enough μ. Here, we present the proof of part 2 (applying to large enough μ), which is very similar to the proof of part 1 given the symmetry of the problem in μ around $\mu = \frac{1}{2}$.

Consider $\mu > \bar{\mu}$ from Lemma 1. Since $\Pr(Piv|q,1) = 0$ and $\Pr(Piv|q,\mu_s)$ is strictly decreasing and strictly convex in μ_s for $\mu_s \in (\bar{\mu}, 1)$, then the concave closure of $\Pr(Piv|q,\mu_s)$ is linear in the neighborhood of $\mu_s = 1$, and the optimal recommendation design for $\mu \in (\bar{\mu}, 1)$ takes the following form: signal $s = 0$ induces belief μ_0 and signal $s = 1$ induces belief $\mu_1 = 1$, where μ_0 is one that maximizes

$$\Pr(Piv|q,\mu_0)\,\tau_0 + \Pr(Piv|q,1)\,\tau_1$$

subject to the Bayes Plausibility constraint $\mu_1\tau_1 + \mu_0\tau_0 = \mu$. Since $\mu_1 = 1$ and $\tau_1 = 1 - \tau_0$, the latter implies $\tau_0 = \frac{1-\mu}{1-\mu_0}$, and since $\Pr(Piv|q,1) = 0$, the average probability of a split vote is $\Pr(Piv|q,\mu_0)\,\frac{1-\mu}{1-\mu_0}$, and μ_0 solves

$$\mu_0 = \arg\max_m \frac{\Pr(Piv|q,m)}{1-m}(1-\mu)\,. \tag{47}$$

The point μ_0 is one where the linear function that starts at $(1,0)$ is tangent to the function $\Pr(Piv|q,\cdot)$. To find μ_0, we substitute (21) into (47) and get

$$\frac{\Pr(Piv|q,m)}{1-m}(1-\mu_0) = 2C_{N-1}^{\frac{N-1}{2}}\left(\varrho_0(q,m)\left(1-\varrho_0(q,m)\right)\right)^{\frac{N-1}{2}}(1-\mu)\,,$$

where $\varrho_0(q,m)$ is given by (24). Hence, μ_0 solves

$$\max_m \left(\varrho_0(q,m)\left(1-\varrho_0(q,m)\right)\right)^{\frac{N-1}{2}}\,. \tag{48}$$

Therefore, if feasible, the optimal m is such that $\varrho_0(q,m) = \frac{1}{2}$. In other words, the probability of a shareholder voting in favor conditional on a controversial recommendation (i.e., $s = 0$) and the state being in line with the recommendation ($\theta = 0$), is exactly 50%. If $m = \frac{1}{2}$, then $\pi(q,m) = \frac{1}{2}$, so $\varrho_0(q,m) = \frac{1-q}{2} < \frac{1}{2}$. Hence, $m = \frac{1}{2}$ does not solve (48) if there exists $m \in \left(\frac{1}{2},\mu\right)$ that satisfies $\varrho_0(q,m) = \frac{1}{2}$. Consider $m \neq \frac{1}{2}$. Then, using (4), $\varrho_0(q,m) = \frac{1}{2}$ is equivalent to

$$\frac{z(1-2q)-1+\sqrt{(z-1)^2+4q^2z}}{2(z-1)} = \frac{1}{2}$$
$$\Leftrightarrow z^2\left(1-4q^2\right) + z\left(4q^2-2\right) + 1 = 0 \Leftrightarrow (z-1)\left(z-\frac{1}{1-4q^2}\right) = 0,$$

where $z \equiv \left(\frac{m}{1-m}\right)^{\frac{2}{N-1}}$. Since $m \neq \frac{1}{2}$, the only root is $z = \frac{1}{1-4q^2}$. Equating it to $\left(\frac{\mu_0}{1-\mu_0}\right)^{\frac{2}{N-1}}$ implies that the optimal posterior is

$$\mu_0(q) = \frac{1}{1+\left(1-4q^2\right)^{\frac{N-1}{2}}}\,. \tag{49}$$

Note that $\mu_0 > \frac{1}{2}$. It follows that the optimal recommendation induces beliefs $\mu_0 \in \left(\frac{1}{2}, \mu\right)$ given by (49) and $\mu_1 = 1$. The average probability of a shareholder being pivotal given this recommendation design is $2C_{N-1}^{\frac{N-1}{2}} \left(\frac{1}{4}\right)^{\frac{N-1}{2}} (1-\mu) = (1-\mu) C_{N-1}^{\frac{N-1}{2}} 2^{2-N}$. The proof that this recommendation is indeed optimal for any $\mu \geq \mu_0(q)$ is similar to the corresponding proof for $\mu \leq \mu_1(q)$: (21) implies that for any (μ_l, μ_h) such that $\mu_l < \mu < \mu_h$, we have

$$\frac{\mu_h - \mu}{\mu_h - \mu_l} \Pr(Piv|q, \mu_l) + \frac{\mu - \mu_l}{\mu_h - \mu_l} \Pr(Piv|q, \mu_h) \leq (1-\mu) C_{N-1}^{\frac{N-1}{2}} 2^{2-N},$$

as required.

A.3 Proof of Proposition 8

Recall that we focus on symmetric equilibria, and also on equilibria in undominated strategies at the voting stage. Consider any public recommendation design $\mathcal{S}$. We will show that for any realization s inducing posterior μ_s and any expected fraction of subscribers q, a shareholder's willingness to pay for an imperfectly informative report is weakly lower than for a fully informative report. With a slight abuse of notation, we will use $\Pr(\cdot|\mu_s)$ to denote the conditioning based on the realization s and $\Pr(\cdot|\mu_s, q)$ to denote the conditioning based on the realization s when the probability of subscribing is q. We also denote

$$L(x) \equiv C_{N-1}^{\frac{N-1}{2}} (x(1-x))^{\frac{N-1}{2}},$$

which captures the probability of a shareholder being pivotal if other shareholders vote for the proposal with probability x.

If $\mu_s = 0$ or 1, the value of the report is zero, regardless of its information content. Consider $\mu_s \in (0,1)$. The proof consists of the following steps.

1: It is sufficient to focus on binary signals, $R = \{0, 1\}$**.**

Consider an arbitrary report $\mathcal{R}$, and let $W(\mathcal{R}, \mu_s, q)$ denote the value of the report (divided by v_i) for shareholder i, conditional on s and given q. Note that when deciding how to vote, a subscribing shareholder does not learn any additional information from the event of being pivotal: this is because all shareholders' votes are based on r and/or s, and the subscribing shareholder knows both of them. Hence, the subscribers only condition on r and s when deciding how to vote.

1.1: Breaking down R **into subsets.** Divide the set of signals R into three subsets:

$$R_0 \equiv \left\{r \in R : \Pr(\theta = 1|r, s) < \frac{1}{2}\right\}, \tag{50}$$

$$R_1 \equiv \left\{r \in R : \Pr(\theta = 1|r, s) > \frac{1}{2}\right\}, \tag{51}$$

$$R_m \equiv \left\{r \in R : \Pr(\theta = 1|r, s) = \frac{1}{2}\right\}. \tag{52}$$

Since we focus on equilibria in undominated strategies, R_0 (R_1) is the set of signals in the report that induce all subscribers to vote "against" ("for") with probability one, and R_m is the set of signals for which the subscribers are indifferent between voting "for" and "against." Note that the value of the report conditional on $r \in R_m$ is zero: because a subscriber is indifferent between voting for and against conditional on such r and being pivotal, he believes that each state is equally likely, so any vote brings the same value. It follows that all else equal, $W(\mathcal{R}, \mu_s, q)$ is higher if set R_m is empty. Hence, we can focus on reports where $\Pr(r \in R_0|\mu_s) + \Pr(r \in R_1 \mu_s) = 1$.

1.2: Non-subscribing shareholders mix in equilibrium. Suppose that non-subscribing shareholders vote for the proposal with probability π. We show that $\pi \in (0,1)$. Indeed, suppose that $\pi = 1$. Since $q \in (0,1)$, then $\Pr(Piv|r \in R_1, s) = 0$ and $\Pr(Piv|r \in R_0, s) > 0$. Hence, $\Pr(\theta = 1|Piv, s) = \Pr(\theta = 1|r \in R_0, s) < \frac{1}{2}$, where the inequality follows from (50). But then, since we focus on weakly undominated strategies, the shareholder must vote against, which contradicts $\pi = 1$. Similarly, suppose that $\pi = 0$. Then $\Pr(Piv|r \in R_0, s) = 0$ and $\Pr(Piv|r \in R_1, s) > 0$. Hence, $\Pr(\theta = 1|Piv, s) = \Pr(\theta = 1|r \in R_1, s) > \frac{1}{2}$, where the inequality follows from (51). But then, since we focus on weakly undominated strategies, the shareholder must vote in favor, which contradicts $\pi = 0$. Thus, indeed, $\pi \in (0,1)$, and we find π below.

1.3: Value from the report. Since $q \in (0,1)$ and non-subscribing shareholders randomize, then for any $\mu_s \in (0,1)$, a non-subscribing shareholder is pivotal with a strictly positive probability. Hence, to find it optimal to randomize, he must be indifferent between voting for and against conditional on s and being pivotal, i.e., $\Pr(\theta = 1|\mu_s, Piv) = \frac{1}{2}$. Then, $\Pr(d = \theta|\mu_s, Piv) = \frac{1}{2}$. Therefore, repeating the derivations in Section A.8 of the Online Appendix,

$$\begin{aligned}
W(\mathcal{R}, \mu_s, q) &= \Pr(Piv|q, \mu_s)\left[\Pr(d = \theta|\mathcal{R}, \mu_s, Piv) - \Pr(d = \theta|\mu_s, Piv)\right] \\
&= \Pr(Piv|q, \mu_s)\left[\Pr(d = \theta|\mathcal{R}, \mu_s, Piv) - \frac{1}{2}\right] \\
&= \sum_{r \in R} \Pr(r|\mu_s)\Pr(Piv|q, r, \mu_s)\left(\Pr(d = \theta|r, \mu_s, Piv) - \frac{1}{2}\right) \\
&= \sum_{r \in R_0} \Pr(r|\mu_s)\Pr(Piv|q, r, \mu_s)\left(\Pr(\theta = 0|r, \mu_s) - \frac{1}{2}\right) \\
&\quad + \sum_{r \in R_1} \Pr(r|\mu_s)\Pr(Piv|q, r, \mu_s)\left(\Pr(\theta = 1|r, \mu_s) - \frac{1}{2}\right),
\end{aligned}$$

where the last equality uses the fact that R_m is empty and the fact that $\Pr(\theta = 1|r, \mu_s, Piv) = \Pr(\theta = 1|r, \mu_s)$ because, as discussed above, there is no added information from the event of being pivotal conditional on observing r.

1.4: Focusing on binary signals. Since we focus on symmetric equilibria, then for any $r \in R_0$ the voting strategy of subscribers is the same, and hence $\Pr(Piv|q, r_1, \mu_s) = \Pr(Piv|q, r_2, \mu_s)$

for any $r_1, r_2 \in R_0$. Similarly, $\Pr(Piv|q, r_1, \mu_s) = \Pr(Piv|q, r_2, \mu_s)$ for any $r_1, r_2 \in R_1$. Thus,

$$\begin{aligned} W(\mathcal{R}, \mu_s, q) &= \Pr(Piv|q, r \in R_0, \mu_s)\Pr(r \in R_0|\mu_s)\left(\Pr(\theta = 0|r \in R_0, \mu_s) - \frac{1}{2}\right) \\ &\quad + \Pr(Piv|q, r \in R_1, \mu_s)\Pr(r \in R_1|\mu_s)\left(\Pr(\theta = 1|r \in R_1, \mu_s) - \frac{1}{2}\right). \end{aligned} \tag{53}$$

Notice that it is without loss of generality to combine all $r \in R_0$ into one signal, denoted $r = 0$, and all $r \in R_1$ into the other signal, denoted $r = 1$. Thus, we can focus on binary signals, as required.

2: An informative report is optimal.

2.1: Finding the strategy of non-subscribers.

Denote

$$\begin{aligned} p_0 &\equiv \Pr(\theta = 0|r = 0, \mu_s), \\ p_1 &\equiv \Pr(\theta = 1|r = 1, \mu_s). \end{aligned}$$

Then (50) and (51) imply

$$p_0 > \frac{1}{2} \text{ and } p_1 > \frac{1}{2}. \tag{54}$$

Since R_m is empty, we have:

$$\begin{gathered} p_1 \Pr(r = 1|\mu_s) + (1 - p_0)\Pr(r = 0|\mu_s) = \mu_s, \\ \Pr(r = 1|\mu_s) + \Pr(r = 0|\mu_s) = 1, \end{gathered} \tag{55}$$

where the top equation follows from Bayes' rule. Solving (55), we get:

$$\begin{gathered} \Pr(r = 1|\mu_s) = \tfrac{\mu_s - (1 - p_0)}{p_1 + p_0 - 1}, \\ \Pr(r = 0|\mu_s) = \tfrac{p_1 - \mu_s}{p_1 + p_0 - 1}. \end{gathered} \tag{56}$$

We next find π. Using Bayes rule,

$$\Pr(\theta = 1|s, Piv) = \frac{\Pr(Piv|\theta = 1, s)\mu_s}{\Pr(Piv|\theta = 1, s)\mu_s + \Pr(Piv|\theta = 0, s)(1 - \mu_s)}. \tag{57}$$

Recall that non-subscribing shareholders must be indifferent between voting for and against, i.e., $\Pr(\theta = 1|s, Piv) = \frac{1}{2}$. Together with (57), this implies

$$\Pr(Piv|\theta = 1, s)\mu_s = \Pr(Piv|\theta = 0, s)(1 - \mu_s),$$

or equivalently,

$$\begin{aligned}&\begin{pmatrix}\Pr\left(Piv|\theta=1,s,r=1\right)\Pr\left(r=1|\theta=1,s\right)\\+\Pr\left(Piv|\theta=1,s,r=0\right)\Pr\left(r=0|\theta=1,s\right)\end{pmatrix}\mu_s\\=&\begin{pmatrix}\Pr\left(Piv|\theta=0,s,r=1\right)\Pr\left(r=1|\theta=0,s\right)\\+\Pr\left(Piv|\theta=0,s,r=0\right)\Pr\left(r=0|\theta=0,s\right)\end{pmatrix}\left(1-\mu_s\right).\end{aligned}\tag{58}$$

Using Bayes' rule and (56),

$$\begin{aligned}\Pr\left(r=1|\theta=1,s\right)&=\frac{\Pr\left(\theta=1|r=1,s\right)\Pr\left(r=1|s\right)}{\Pr\left(\theta=1|s\right)}=\frac{p_1}{\mu_s}\Pr\left(r=1|s\right)=\frac{p_1}{\mu_s}\frac{\mu_s-(1-p_0)}{p_1+p_0-1},\\\Pr\left(r=0|\theta=1,s\right)&=\frac{\Pr\left(\theta=1|r=0,s\right)\Pr\left(r=0|s\right)}{\Pr\left(\theta=1|\mu_s\right)}=\frac{1-p_0}{\mu_s}\Pr\left(r=0|s\right)=\frac{1-p_0}{\mu_s}\frac{p_1-\mu_s}{p_1+p_0-1},\\\Pr\left(r=1|\theta=0,s\right)&=\frac{\Pr\left(\theta=0|r=1,s\right)\Pr\left(r=1|s\right)}{\Pr\left(\theta=0|\mu_s\right)}=\frac{1-p_1}{1-\mu_s}\Pr\left(r=1|s\right)=\frac{1-p_1}{1-\mu_s}\frac{\mu_s-(1-p_0)}{p_1+p_0-1},\\\Pr\left(r=0|\theta=0,s\right)&=\frac{\Pr\left(\theta=0|r=0,s\right)\Pr\left(r=0|s\right)}{\Pr\left(\theta=0|s\right)}=\frac{p_0}{1-\mu_s}\Pr\left(r=0|s\right)=\frac{p_0}{1-\mu_s}\frac{p_1-\mu_s}{p_1+p_0-1}.\end{aligned}$$

Plugging this into (58), we get:

$$\begin{aligned}&\left(L\left(q+(1-q)\pi\right)\frac{p_1}{\mu_s}\frac{\mu_s-(1-p_0)}{p_1+p_0-1}+L\left((1-q)\pi\right)\frac{1-p_0}{\mu_s}\frac{p_1-\mu_s}{p_1+p_0-1}\right)\mu_s\\=&\left(L\left(q+(1-q)\pi\right)\frac{1-p_1}{1-\mu_s}\frac{\mu_s-(1-p_0)}{p_1+p_0-1}+L\left((1-q)\pi\right)\frac{p_0}{1-\mu_s}\frac{p_1-\mu_s}{p_1+p_0-1}\right)\left(1-\mu_s\right),\end{aligned}$$

or equivalently,

$$\begin{aligned}&L\left(q+(1-q)\pi\right)\left[p_1\frac{\mu_s-(1-p_0)}{p_1+p_0-1}-(1-p_1)\frac{\mu_s-(1-p_0)}{p_1+p_0-1}\right]\\=&L\left((1-q)\pi\right)\left[p_0\frac{p_1-\mu_s}{p_1+p_0-1}-(1-p_0)\frac{p_1-\mu_s}{p_1+p_0-1}\right]\Leftrightarrow\\&L\left(q+(1-q)\pi\right)\left[(2p_1-1)\mu_s+p_0(2p_1-1)+1-2p_1\right]\\=&L\left((1-q)\pi\right)\left[(2p_0-1)p_1+\mu_s(1-2p_0)\right],\end{aligned}$$

which gives:

$$L\left(q+(1-q)\pi\right)\left(p_1-\frac{1}{2}\right)\left(\mu_s-(1-p_0)\right)=L\left((1-q)\pi\right)\left(p_0-\frac{1}{2}\right)\left(p_1-\mu_s\right).\tag{59}$$

Next, (59) is equivalent to

$$\begin{aligned}\frac{L\left(q+\left(1-q\right)\pi\right)}{L\left(\left(1-q\right)\pi\right)} &= \frac{p_0-\frac{1}{2}}{p_1-\frac{1}{2}}\frac{p_1-\mu_s}{\mu_s-\left(1-p_0\right)}=\frac{p_1-\mu_s}{p_1-\frac{1}{2}}\frac{p_0-\frac{1}{2}}{p_0+\mu_s-1}\Leftrightarrow &(60)\\ \frac{\pi\left(1-\left(1-q\right)\pi\right)}{\left(q+\left(1-q\right)\pi\right)\left(1-\pi\right)} &= \left(\frac{p_1-\frac{1}{2}}{p_1-\mu_s}\frac{p_0+\mu_s-1}{p_0-\frac{1}{2}}\right)^{\frac{2}{N-1}}\equiv\tilde{z}_s &(61)\end{aligned}$$

Denoting the right-hand side of (61) by $\tilde{z}_s$, we note that (61) gives exactly the same equation on π as in the proof of Proposition 2, i.e., $w\left(\pi,\tilde{z}_s\right)=0$, where $w\left(\pi,\cdot\right)$ is given by (16). The proof of Proposition 2 shows that for any $\tilde{z}_s$, there is a unique root in $(0,1)$, which equals $\frac{1}{2}$ if $\tilde{z}_s=1$, and is given by

$$\pi\left(q,\tilde{z}_s\right)=\frac{\tilde{z}_s\left(1-2q\right)-1+\sqrt{\left(\tilde{z}_s-1\right)^2+4q^2\tilde{z}_s}}{2\left(\tilde{z}_s-1\right)\left(1-q\right)}$$

if $\tilde{z}_s\neq 1$, where $\pi\left(q,\tilde{z}_s\right)$ increases in $\tilde{z}_s$ and $\pi\left(q,\tilde{z}_s\right)>\frac{1}{2}$ if and only if $\tilde{z}_s>1$. Note that $\tilde{z}_s>1$ if and only if $\mu_s>\frac{1}{2}$, and

$$\begin{aligned}\frac{d}{dp_1}\left(\frac{p_1-\mu_s}{p_1-\frac{1}{2}}\right) &= \frac{\mu_s-\frac{1}{2}}{\left(p_1-\frac{1}{2}\right)^2},\\ \frac{d}{dp_0}\left(\frac{p_0-\frac{1}{2}}{p_0+\mu_s-1}\right) &= \frac{\mu_s-\frac{1}{2}}{\left(p_0+\mu_s-1\right)^2}.\end{aligned}\qquad(62)$$

2.2: Showing that $p_0=p_1=1$. Using (53) and (56),

$$\begin{aligned}&W\left(\mathcal{R},\mu_s,q\right)=\\ &=L\left(\left(1-q\right)\pi\right)\Pr\left(r=0|\mu_s\right)\left(p_0-\frac{1}{2}\right)+L\left(q+\left(1-q\right)\pi\right)\Pr\left(r=1|\mu_s\right)\left(p_1-\frac{1}{2}\right) &(63)\\ &=L\left(\left(1-q\right)\pi\right)\frac{p_1-\mu_s}{p_1+p_0-1}\left(p_0-\frac{1}{2}\right)+L\left(q+\left(1-q\right)\pi\right)\frac{\mu_s-\left(1-p_0\right)}{p_1+p_0-1}\left(p_1-\frac{1}{2}\right) &(64)\end{aligned}$$

where $\pi\in(0,1)$ is the probability with which each non-subscriber votes for the proposal found above (we omit its dependence on μ_s and q for brevity). Plugging (59) into (64), we get

$$\begin{aligned}W\left(\mathcal{R},\mu_s,q\right) &= L\left(\left(1-q\right)\pi\right)\frac{p_1-\mu_s}{p_1+p_0-1}\left(2p_0-1\right) &(65)\\ &= L\left(q+\left(1-q\right)\pi\right)\frac{\mu_s-\left(1-p_0\right)}{p_1+p_0-1}\left(2p_1-1\right). &(66)\end{aligned}$$

Note that the function $L\left(x\right)$ increases in x for $x\in\left(0,\frac{1}{2}\right)$ and decreases for $x\in\left(\frac{1}{2},1\right)$.

2.2.1: Case of $\mu_s>\frac{1}{2}$. If $\mu_s>\frac{1}{2}$, then $\tilde{z}_s>1$, so $\pi>\frac{1}{2}$. Hence, $q+\left(1-q\right)\pi>q+\left(1-q\right)\frac{1}{2}>\frac{1}{2}$, and hence $L\left(q+\left(1-q\right)\pi\right)$ decreases in π in this range. Consider an increase in p_1. Then (62) implies that $\frac{p_1-\mu_s}{p_1-\frac{1}{2}}$ increases, so $\tilde{z}_s$ decreases, and hence π decreases. A reduction in $\pi>\frac{1}{2}$

increases $L(q+(1-q)\pi)$. Moreover, $\frac{d}{dp_1}\left[\frac{2p_1-1}{p_1+p_0-1}\right] = \frac{2p_0-1}{(p_1+p_0-1)^2} > 0$ by (54). Hence, (66) implies that $W(\mathcal{R},\mu_s,q)$ increases. Hence, $p_1=1$ is optimal.

Similarly, consider an increase in p_0. Then (62) implies that $\frac{p_0-\frac{1}{2}}{p_0+\mu_s-1}$ increases, so $\tilde{z}_s$ decreases, and hence π decreases. A reduction in $\pi > \frac{1}{2}$ increases $L(q+(1-q)\pi)$. Moreover $\frac{d}{dp_0}\left[\frac{\mu_s-(1-p_0)}{p_1+p_0-1}\right] = \frac{p_1-\mu_s}{(p_1+p_0-1)^2}$. Given that $p_1=1$ is optimal, this derivative is positive, and hence (66) implies that $W(\mathcal{R},\mu_s,q)$ increases. Thus, $p_0=1$ is also optimal.

2.2.2: Case of $\mu_s < \frac{1}{2}$. If $\mu_s < \frac{1}{2}$, then $\tilde{z}_s < 1$, so $\pi < \frac{1}{2}$. Hence, $(1-q)\pi < \frac{1-q}{2} < \frac{1}{2}$, and hence $L((1-q)\pi)$ increases in π in this range. Consider an increase in p_0. Then (62) implies that $\frac{p_0-\frac{1}{2}}{p_0+\mu_s-1}$ decreases, so $\tilde{z}_s$ increases, and hence π increases. An increase in $\pi < \frac{1}{2}$ increases $L((1-q)\pi)$. Moreover, $\frac{d}{dp_0}\left[\frac{2p_0-1}{p_1+p_0-1}\right] = \frac{2p_1-1}{(p_1+p_0-1)^2} > 0$ by (54). Hence, (65) implies that $W(\mathcal{R},\mu_s,q)$ increases. Hence, $p_0=1$ is optimal.

Similarly, consider an increase in p_1. Then (62) implies that $\frac{p_1-\mu_s}{p_1-\frac{1}{2}}$ decreases, so $\tilde{z}_s$ increases, and hence π increases. An increase in $\pi < \frac{1}{2}$ increases $L((1-q)\pi)$. Moreover $\frac{d}{dp_1}\left[\frac{p_1-\mu_s}{p_1+p_0-1}\right] = \frac{p_0-1+\mu_s}{(p_1+p_0-1)^2}$. Given that $p_0=1$ is optimal, this derivative is positive, and hence (65) implies that $W(\mathcal{R},\mu_s,q)$ increases. Thus, $p_1=1$ is also optimal.

2.2.3: Case of $\mu_s = \frac{1}{2}$. If $\mu_s = \frac{1}{2}$, then $\tilde{z}_s = \frac{1}{2}$ and $\pi = \frac{1}{2}$, so

$$W(\mathcal{R},\mu_s,q) = \frac{1}{2}L\left((1-q)\frac{1}{2}\right)\frac{(2p_1-1)(2p_0-1)}{p_1+p_0-1}.$$

Here $\frac{d}{dp_1}\left[\frac{2p_1-1}{p_1+p_0-1}\right] = \frac{2p_0-1}{(p_1+p_0-1)^2} > 0$, so $p_1=1$ is optimal, and $\frac{d}{dp_0}\left[\frac{2p_0-1}{p_1+p_0-1}\right] = \frac{2p_1-1}{(p_1+p_0-1)^2} > 0$, so $p_0=1$ is also optimal.

Hence, for all μ_s, it is optimal to have $p_0=p_1=1$, i.e., a fully informative report.

A.4 Proof of Proposition 9

To prove part (i), we use the insight in Theorem 2 of McLennan (1998) that in a pure common value environment, a symmetric mixed strategy profile that maximizes the players' expected utility in the class of symmetric mixed strategy profiles must be a Nash equilibrium. Although our game is not a pure common value environment since shareholders differ in the extent to which they care about the proposal, v_i, the voting subgame is equivalent to a pure common value environment in which all shareholders care about the probability of a correct decision and learn the state with an exogenous probability. Formally, consider a voting game of N shareholders with the following ingredients. With probability q^*, a shareholder learns the state and votes $v_i=\theta$. With probability $1-q^*$, a shareholder does not learn the state and only learns a public signal realization s, induced by policy $\mathcal{S}^*$. Each shareholder maximizes the probability of a correct decision. This is a symmetric voting environment in which all voters have common interests. By Proposition 2, this voting game has a unique equilibrium in the class of symmetric mixed strategy equilibria, and in this equilibrium, non-subscribing shareholders vote "for" with probability $\pi(q^*, \mu_s)$, given by (4). Combining this result with Theorem 2 in McLennan (1998) implies that this equilibrium must maximize the expected probability of a correct decision, $\Pr(d = \theta)$, in the class of all symmetric mixed strategies, including those that do not depend on the public signal s. In other words, $\Pr(d = \theta)$ is higher under $\pi^*(q^*, \mu, s) \equiv \pi(q^*, \mu_s)$ than under $\pi(q^*, \mu)$, which is the equilibrium mixed strategy profile when the public signal is uninformative. Since $\mathcal{S}^*$ is not fully uninformative, i.e., $\Pr(\mu_s \neq \mu) > 0$, then $\Pr(\pi^*(q^*, \mu, s) \neq \pi(q^*, \mu_s)) > 0$, and hence the comparison is strict.

We next prove part (ii). Recall that we assume that the conditions of Proposition 6 are satisfied, i.e., $\mu < \mu_1(q^*)$ or $\mu > \mu_0(q^*)$. Consider $\mu < \mu_1(q^*)$ (the case of $\mu > \mu_0(q^*)$ is analogous). Consider the optimal q_{uninf}. Since the probability of a shareholder being pivotal under an uninformative recommendation is $\Pr(Piv|q, \mu)$,

$$q_{uninf} = \arg\max_q qH^{-1}(1-q)\Pr(Piv|q,\mu). \tag{67}$$

The derivative of the objective function is positive if and only if

$$\left[qH^{-1}(1-q)\right]' + qH^{-1}(1-q)\frac{\frac{\partial \Pr(Piv|q,\mu)}{\partial q}}{\Pr(Piv|q,\mu)} > 0. \tag{68}$$

The proof of Proposition 7 shows that the assumption of an increasing hazard rate of $H(\cdot)$ guarantees that

$$\left[qH^{-1}(1-q)\right]' \geq 0 \text{ for all } q \leq q^*, \tag{69}$$

where $q^* = \arg\max_q qH^{-1}(1-q)$. Next, we prove that $\frac{\partial \Pr(Piv|q,\mu)}{\partial q} > 0$ for all $q \leq q^*$. For any q and $\mu \in (0,1)$, using (22), we have:

$$\Pr(Piv|q,\mu) = 2\mu C_{N-1}^{\frac{N-1}{2}}\left(\varrho_1(q,\mu)(1-\varrho_1(q,\mu))\right)^{\frac{N-1}{2}},$$

where

$$\varrho_1(q,\mu) = q + (1-q)\pi(q,\mu) = q + \frac{z(1-2q) - 1 + \sqrt{(z-1)^2 + 4q^2 z}}{2(z-1)}.$$

Hence, $\frac{\partial \Pr(Piv|q,\mu)}{\partial q} > 0$ if and only if

$$\frac{\partial}{\partial q}\left[\varrho_1(q,\mu)(1-\varrho_1(q,\mu))\right] > 0 \Leftrightarrow \frac{\partial \varrho_1(q,\mu)}{\partial q}(1 - 2\varrho_1(q,\mu)) > 0,$$

where

$$\frac{\partial \varrho_1(q,\mu)}{\partial q} = 1 + \frac{-2z + \frac{8qz}{2\sqrt{(z-1)^2+4q^2z}}}{2(z-1)} = \frac{\sqrt{(z-1)^2 + 4q^2z} - \sqrt{4q^2z^2}}{(1-z)\sqrt{(z-1)^2+4q^2z}} > 0$$

since $z > 1$ (which, in turn, follows from $\mu < \mu_1(q^*) < \frac{1}{2}$). Hence, if $\mu < \mu_1(q^*)$, then

$\frac{\partial \Pr(Piv|q,\mu)}{\partial q} > 0$ if and only if $\varrho_1(q,\mu) = q + (1-q)\pi(q,\mu) < \frac{1}{2}$. As shown in the proof of Proposition 4, at the optimal controversial recommendation, we have $\varrho_1(q,\mu_1(q)) = \frac{1}{2}$. Since $\pi(q,\mu)$ is increasing in μ, then $\varrho_1(q,\mu)$ is also increasing in μ. Then, for any $q \leq q^*$, we have $\mu < \mu_1(q^*) \leq \mu_1(q)$, and hence $\varrho_1(q,\mu) < \varrho_1(q,\mu_1(q)) = \frac{1}{2}$. Combined, we have

$$\frac{\partial \Pr(Piv|q,\mu)}{\partial q} > 0 \text{ for all } q \leq q^*. \tag{70}$$

Combining (68), (69), and (70), we conclude that the derivative of the advisor's objective function in (67) is strictly positive for all $q \leq q^*$. Hence, $q_{uninf} > q^*$.

A.5 Derivations for the example in Section 4.1

Suppose that $v_i \in \{v_L, v_H\}$ with $v_H > v_L$ and $\Pr(v_i = v_H) = p$. All the arguments in the basic model continue to hold for this degenerate distribution. For this distribution, we have:

$$H(v) = \begin{cases} 0, & v < v_L \\ 1-p, & v \in [v_L, v_H) \\ 1, & v \geq v_H \end{cases}$$

Suppose μ is sufficiently small (the case of large μ, which is discussed in the paper, is equivalent by symmetry of the problem in μ around $\frac{1}{2}$).

No ban on public recommendations. First, consider the case in which there is no ban on public recommendations. As the arguments in the basic model show, in the range of (q,μ) for which a partially informative recommendation of the form in Proposition 4 is optimal, the average probability of a shareholder being pivotal is the same and equals $\mu C_{N-1}^{\frac{N-1}{2}} 2^{2-N}$ (denote it ζ). Hence, in this range, the advisor's optimal q maximizes $qH^{-1}(1-q)$, which, in the two-type case, means choosing between: 1) selling only to high types, in which case the fee is $\frac{v_H}{2}\zeta$, and profits are $p\frac{v_H}{2}\zeta$ and 2) selling also to some low types, for a total fraction q, in which case the fee is $\frac{v_L}{2}\zeta$, and profits are $q\frac{v_L}{2}\zeta$. If v_H is sufficiently high, then $pv_H > qv_L$, and hence it is optimal for the advisor to only sell the report to high types ($q = p$). In particular, recall from Proposition 7 that a partially informative recommendation of the form in Proposition 4 is optimal only if $q < 0.5$. Hence, for any $v_H \geq 5$, we have $pv_H \geq 0.5 > qv_L$ for any such q.

We conclude that without a ban, if $v_H \geq 5$, then $q = p$, i.e., only high types subscribe to the research report. The optimal recommendation design for a fixed probability of subscribing is given by Proposition 4. Hence, if $\mu < \mu_1(p)$, where $\mu_1(\cdot)$ is defined in Proposition 4, then the optimal recommendation induces posterior beliefs 0 and $\mu_1(p)$, and the average probability that a shareholder is pivotal is $\mu C_{N-1}^{\frac{N-1}{2}} 2^{2-N}$ (as shown in the proof of Proposition 4). Using (9), the price that the seller charges is

$$f = v_H \frac{1}{2}\left[\mu C_{N-1}^{\frac{N-1}{2}} 2^{2-N}\right] = v_H \mu C_{N-1}^{\frac{N-1}{2}} 2^{1-N},$$

and the expected profits of the advisor are

$$\Pi_{no\ ban} = Nfp = Nv_H \mu C_{N-1}^{\frac{N-1}{2}} 2^{1-N} p.$$

The probability of a correct decision is:

$$V_{no\ ban} = \Pr\left(d = \theta | p, \mathcal{S}^*\right) = \mu \Pr\left(d = 1 | p, \mathcal{S}^*, \theta = 1\right) + (1-\mu) \Pr\left(d = 0 | p, \mathcal{S}^*, \theta = 0\right),$$

where $\Pr\left(d = 1 | p, \mathcal{S}^*, \theta = 1\right) = \sum_{k=\frac{N+1}{2}}^{N} C_N^k \left(\frac{1}{2}\right)^N$ because, as shown in the proof of Proposition 4, $\varrho_1\left(p, \mu_1\left(p\right)\right) = \frac{1}{2}$. Next,

$$\begin{aligned} \Pr\left(d = 0 | p, \mathcal{S}^*, \theta = 0\right) &= \Pr\left(d = 0 | p, \mathcal{S}^*, \theta = 0, s = 0\right) \Pr\left(s = 0 | \theta = 0\right) \\ &\quad + \Pr\left(d = 0 | p, \mathcal{S}^*, \theta = 0, s = 1\right) \Pr\left(s = 1 | \theta = 0\right). \end{aligned}$$

Consider the first term. Since $s = 0$ induces belief $\mu_0 = 0$ and is given with probability $1 - \frac{\mu}{\mu_1(p)}$, we have

$$\begin{aligned} \Pr\left(d = 0 | p, \mathcal{S}^*, \theta = 0, s = 0\right) \Pr\left(s = 0 | \theta = 0\right) &= \Pr\left(s = 0 | \theta = 0\right) \\ = \frac{\Pr\left(\theta = 0 | s = 0\right) \Pr\left(s = 0\right)}{\Pr\left(\theta = 0\right)} &= \frac{1 - \frac{\mu}{\mu_1(p)}}{1-\mu}. \end{aligned}$$

Consider the second term. Recall from the proof of Proposition that $\varrho_0\left(p, \mu_1\left(p\right)\right) = \left(1-p\right) \pi\left(p, \mu_1\left(p\right)\right)$, where $\mu_1\left(p\right)$ satisfies $\left(1-p\right)\left(1 - \pi\left(p, \mu_1\left(p\right)\right)\right) = \frac{1}{2}$. Hence,

$$1 - \varrho_0\left(p, \mu_1\left(p\right)\right) = 1 - \left(1-p\right) \pi\left(p, \mu_1\left(p\right)\right) = p + \left(1-p\right)\left(1 - \pi\left(p, \mu_1\left(p\right)\right)\right) = p + \frac{1}{2}$$

and $\varrho_0\left(p, \mu_1\left(p\right)\right) = \frac{1}{2} - p$. Since $s = 1$ induces belief $\mu_1\left(p\right)$ and is given with probability $\frac{\mu}{\mu_1(p)}$, we have

$$\begin{aligned} &\Pr\left(d = 0 | p, \mathcal{S}^*, \theta = 0, s = 1\right) \Pr\left(s = 1 | \theta = 0\right) \\ &= \left(\sum_{k=0}^{\frac{N-1}{2}} C_N^k \left(\varrho_0\left(p, \mu_1\left(p\right)\right)\right)^k \left(1 - \varrho_0\left(p, \mu_1\left(p\right)\right)\right)^{N-k}\right) \frac{\Pr\left(\theta = 0 | s = 1\right) \Pr\left(s = 1\right)}{\Pr\left(\theta = 0\right)} \\ &= \left(\sum_{k=0}^{\frac{N-1}{2}} C_N^k \left(\frac{1}{2} - p\right)^k \left(p + \frac{1}{2}\right)^{N-k}\right) \frac{\left(1 - \mu_1\left(p\right)\right) \frac{\mu}{\mu_1(p)}}{1-\mu}. \end{aligned}$$

Combining, we get:

$$\begin{aligned} V_{no\ ban} &= \mu \sum_{k=\frac{N+1}{2}}^{N} C_N^k \left(\tfrac{1}{2}\right)^N + (1-\mu)\left(\frac{1 - \frac{\mu}{\mu_1(p)}}{1-\mu} + \left(\sum_{k=0}^{\frac{N-1}{2}} C_N^k \left(\tfrac{1}{2} - p\right)^k \left(p + \tfrac{1}{2}\right)^{N-k}\right) \frac{\left(1 - \mu_1(p)\right) \frac{\mu}{\mu_1(p)}}{1-\mu}\right) \\ &= \mu \tfrac{1}{2} + 1 - \tfrac{\mu}{\mu_1(p)} + \left(\sum_{k=0}^{\frac{N-1}{2}} C_N^k \left(\tfrac{1}{2} - p\right)^k \left(p + \tfrac{1}{2}\right)^{N-k}\right) \left(1 - \mu_1\left(p\right)\right) \tfrac{\mu}{\mu_1(p)}. \end{aligned}$$

Ban on public recommendations. Next, consider the case with a ban on public recommendations. There are two possibilities: either the advisor continues to sell to only high types, or it also sells to a fraction of low types. First, consider the case in which the seller sells to some of the low types too. If the optimal recommendation is uninformative, then using (22), a shareholder is pivotal with probability

$$\Pr\left(Piv|q,\mu\right) = 2\mu C_{N-1}^{\frac{N-1}{2}} \left(\left(q + (1-q)\,\pi\left(q,\mu\right)\right)(1-q)\left(1-\pi\left(q,\mu\right)\right)\right)^{\frac{N-1}{2}}.$$

Then, the shareholder of a low type is willing to pay $\frac{v_L}{2}\Pr\left(Piv|q,\mu\right)$, and hence the optimal q solves:

$$q^{**} = \arg\max_{q} q\left(\left(q + (1-q)\,\pi\left(q,\mu\right)\right)(1-q)\left(1-\pi\left(q,\mu\right)\right)\right)^{\frac{N-1}{2}}, \tag{71}$$

where

$$\pi\left(q,\mu\right) = \frac{z(1-2q)-1+\sqrt{(z-1)^2+4q^2z}}{2(z-1)(1-q)} \Leftrightarrow$$
$$q + (1-q)\,\pi\left(q,\mu\right) = \frac{1}{2} + \frac{\sqrt{(z-1)^2+4q^2z}-2q}{2(z-1)},$$

and $z = \left(\frac{\mu}{1-\mu}\right)^{\frac{2}{N-1}}$. Plugging this into (71), we can find q^{**} by solving:

$$q^{**} = \arg\max_{q} q\left(1 - \frac{\left(\sqrt{(z-1)^2+4q^2z}-2q\right)^2}{(z-1)^2}\right)^{\frac{N-1}{2}}.$$

To induce such an expected fraction of subscribers, the proxy advisor needs to charge fee

$$f = \frac{v_L}{2}\Pr\left(Piv|q^{**},\mu\right) = \frac{v_L}{2} C_{N-1}^{\frac{N-1}{2}} \left(\left(q^{**} + (1-q^{**})\,\pi\left(q^{**},\mu\right)\right)(1-q^{**})\left(1-\pi\left(q^{**},\mu\right)\right)\right)^{\frac{N-1}{2}},$$

and the corresponding expected profits are:

$$\Pi_{ban,both\ types} = Nfq^{**} = N\frac{v_L}{2} C_{N-1}^{\frac{N-1}{2}} \left(\left(q^{**} + (1-q^{**})\,\pi\left(q^{**},\mu\right)\right)(1-q^{**})\left(1-\pi\left(q^{**},\mu\right)\right)\right)^{\frac{N-1}{2}} q^{**}.$$

The probability of the correct decision is:

$$V_{ban,both\ types} = \Pr(d=\theta|q^{**},\mu) = \mu \Pr(d=1|q^{**},\mu,\theta=1) + (1-\mu)\Pr(d=0|q^{**},\mu,\theta=0) \quad (72)$$

$$= \mu\left(\sum_{k=\frac{N+1}{2}}^{N} C_N^k \left(q^{**}+(1-q^{**})\pi(q^{**},\mu)\right)^k \left((1-q^{**})(1-\pi(q^{**},\mu))\right)^{N-k}\right) \quad (73)$$

$$+(1-\mu)\left(\sum_{k=0}^{\frac{N-1}{2}} C_N^k \left((1-q^{**})\pi(q^{**},\mu)\right)^k \left(1-(1-q^{**})\pi(q^{**},\mu)\right)^{N-k}\right) \quad (74)$$

$$= \mu\left(\sum_{K=0}^{\frac{N-1}{2}} C_N^K \left(q^{**}+(1-q^{**})\pi(q^{**},\mu)\right)^{N-K} \left((1-q^{**})(1-\pi(q^{**},\mu))\right)^{K}\right)$$

$$+(1-\mu)\left(\sum_{k=0}^{\frac{N-1}{2}} C_N^k \left((1-q^{**})\pi(q^{**},\mu)\right)^k \left(1-(1-q^{**})\pi(q^{**},\mu)\right)^{N-k}\right),$$

where the last equality uses the notation $K = N - k$ in the first term.

Second, consider the case where even with the ban the seller wants to sell to high types only. In this case, the same fraction p of shareholders observe the report (and the state), but all other shareholders are now informed. Part (i) of Proposition 9 then directly implies that the probability of the correct decision, $V_{ban,high\ type}$, is strictly lower than without a ban, $V_{no\ ban}$. The profits of the seller in this case are

$$\Pi_{ban,high\ type} = Nfp = N\frac{v_H}{2}C_{N-1}^{\frac{N-1}{2}}\left((p+(1-p)\pi(p,\mu))(1-p)(1-\pi(p,\mu))\right)^{\frac{N-1}{2}}p,$$

and the probability of the correct decision is given by the sum of (73) and (74) but with q^{**} replaced by p.

Numerically, we find that if $N = 25$ and $\mu = 0.1$ (or $\mu = 0.9$), then $q^{**} = 0.257$. If $v_H = 5$, then under the ban, the advisor's profit from selling to both high and low types is $\Pi_{ban,both\ types} = 0.0927$, whereas its profit from selling only to high types is $\Pi_{ban,high\ type} = 0.0732$. Hence, it is optimal to sell to both types and induce $q^{**} = 0.257$, which results in the probability of a correct decision of $V_{ban,both\ types} = 0.9565$. On the other hand, if $v_H = 7$, then under the ban, the advisor's profit from selling to both high and low types is $\Pi_{ban,both\ types} = 0.0927$, whereas its profit from selling only to high types is $\Pi_{ban,high\ type} = 0.1025$. Hence, it is optimal to sell to only the high types and induce fraction of subscribers $p = 0.1$, which results in the probability of a correct decision of $V_{ban,both\ types} = 0.9031$.

A.6 Proof of Proposition 10

The same arguments as in the proof of Proposition 8 imply that the optimal research report is fully informative about the state. The solution of this variation of the model is similar to the solution of the baseline model with the following modifications. A shareholder whose

$v_i > \hat{v}$ does not subscribe to the report. Let σ denote the probability with which a shareholder subscribes to the report conditional on $v_i < \hat{v}$. Then the probability with which a shareholder learns the state, which we denote by q, is

$$q = \Pr(v_i > \hat{v}) + \sigma \Pr(v_i < \hat{v}) = \chi + (1-\chi)\sigma,$$

or equivalently, $\sigma = \frac{q-\chi}{1-\chi}$ and $1-\sigma = \frac{1-q}{1-\chi}$.

The voting subgame and the information acquisition decisions of shareholders who are not exogenously informed depend on q, the overall probability of other shareholders becoming informed, in exactly the same way as before. In particular, the equilibrium probability that an uninformed shareholder votes for the proposal is $\pi(q, \mu_s)$ given by (4), and the value from the report for a shareholder whose v_i is below $\hat{v}$ is $v_i V(q, \mathcal{S})$, where $V(q, \mathcal{S})$ is given by (7). Therefore,

$$\sigma = \Pr(f/V(q,\mathcal{S}) \le v_i < \hat{v} | v_i < \hat{v}) = \frac{H(\hat{v}) - H(f/V(q,\mathcal{S}))}{H(\hat{v})} = \frac{1 - H(f/V(q,\mathcal{S})) - \chi}{1-\chi}.$$

Hence, we can find the fee that induces expected fraction q of informed shareholders from

$$H(f/V(q,\mathcal{S})) = (1-\chi)(1-\sigma) = 1-q \Leftrightarrow f = V(q,\mathcal{S}) H^{-1}(1-q),$$

as in the basic model. The expected fraction of shareholders subscribing to the report is $\Pr(f/V(q,\mathcal{S}) \le v_i < \hat{v}) = \sigma \Pr(v_i < \hat{v}) = q - \chi$, and thus the expected profit of the advisor is $N(q-\chi) V(q,\mathcal{S}) H^{-1}(1-q)$. Hence, the advisor's problem is now

$$\begin{gathered} \max_{q,S,\{\mu_s,\tau_s\}} (q-\chi) H^{-1}(1-q) \sum_{s\in S} \Pr(Piv|q,\mu_s)\tau_s \\ s.to \ \sum_{s\in S} \mu_s \tau_s = \mu. \end{gathered}$$

As in the basic model, we can decompose this problem into two steps: first, find the optimal recommendation design for a given q, and then, find the optimal q and fee. For a given q, the public recommendation design problem is exactly the same as in the basic model. Thus, the analysis of Section 3.3.1 and Proposition 4 are unchanged. The part that is different is the pricing of information by the advisor. As in the basic model, the average probability of being pivotal is the same for all pairs of (q, μ) for which the optimal recommendation is partially informative and of the form described by Proposition 4. Hence, it is optimal for the advisor to choose

$$q_e^*(\chi) = \arg\max_q (q-\chi) H^{-1}(1-q), \tag{75}$$

as long as $q_e^*(\chi) < \frac{1}{2}$ and priors are sufficiently asymmetric, as in Proposition 6. The cross-partial derivative of the objective function $(q-\chi) H^{-1}(1-q)$ in q and χ is $\frac{\partial}{\partial q}[-H^{-1}(1-q)]$, which is positive because $H^{-1}(\cdot)$ is an increasing function. Hence, by Topkis's theorem, $q_e^*(\chi)$ is increasing in χ. Therefore, the comparative statics of the recommendation design in χ is the same as the comparative statics of the optimal recommendation design in q in the basic model, which is given by Corollary 1. In addition, it follows that $q_e^*(\chi) > q_e^*(0) = q^*$, the equilibrium expected fraction of informed shareholders in the basic model.

A.7 Proof of Proposition 11

For shareholder i, the value of subscribing to the proxy advisor's services is

$$V_i(q,\mathcal{S})+\omega=v_i\cdot V(q,\mathcal{S})+\omega.$$

Given fee $f\geq\omega$, the shareholder becomes the subscriber if and only if $v_i\geq\frac{f-\omega}{V(q,\mathcal{S})}$, which implies

$$f=V(q,\mathcal{S})\,H^{-1}(1-q)+\omega=H^{-1}(1-q)\left(\frac{1}{2}\sum_{s\in S}\Pr(Piv|q,\mu_s)\,\tau_s\right)+\omega.$$

Since the proxy advisor's expected revenue is Nqf, its problem is to choose q and $\mathcal{S}$ to solve:

$$\max_{q,\mathcal{S}}\left\{qH^{-1}(1-q)\left(\sum_{s\in S}\Pr(Piv|q,\mu_s)\,\tau_s\right)+2q\omega\right\}.$$

For a fixed q, the optimization problem of the proxy advisor over $\mathcal{S}$ is the same as in the basic model. Recall that its solution gives the maximum average probability of a shareholder being pivotal of $P(q,\mu)$. Therefore, the only aspect that changes compared to the basic model is the q targeted by the advisor. The optimization problem becomes

$$\max_{q}\left\{qH^{-1}(1-q)\,P(q,\mu)+2q\omega\right\}. \tag{76}$$

Let $q^*(\omega)$ be the solution to this problem. By Topkis's theorem, the sign of $\frac{\partial q^*(\omega)}{\partial\omega}$ coincides with the sign of

$$\frac{\partial^2}{\partial q\partial\omega}\left[qH^{-1}(1-q)\,P(q,\mu)+2q\omega\right]|_{q=\tilde{q}(\omega)}=2.$$

Hence, $q^*(\omega)$ increases in ω. Given an equilibrium increase in the expected fraction of subscribers, the effect on the recommendation design follows from Corollary 1.

A.8 Value of buying the report

We show that for an arbitrary research report $\mathcal{R}$ and recommendation policy $\mathcal{S}$, the value $W_i(\mathcal{R},\mathcal{S})$ of buying the report for shareholder i is $v_i\cdot W(\mathcal{R},\mathcal{S})$, where

$$W(\mathcal{R},\mathcal{S}) = \Pr(Piv)\left[\Pr(d=\theta|Piv,\mathcal{R})-\Pr(d=\theta|Piv,\mathcal{S})\right] \tag{77}$$

$$= \sum_{s\in S}\Pr(s)\Pr(Piv|s)\left[\Pr(d=\theta|Piv,\mathcal{R})-\Pr(d=\theta|Piv,s)\right], \tag{78}$$

and Piv denotes the event that the shareholder is pivotal.

We first show that $W(\mathcal{R},\mathcal{S})=\Pr(d=\theta|\mathcal{R})-\Pr(d=\theta|\mathcal{S})$. Indeed, let $U(\mathcal{R})$ and $U(\mathcal{S})$ denote the shareholder's expected utility (divided by v_i) from knowing the report and from knowing only the recommendation, respectively. Then

$$W(\mathcal{R},\mathcal{S}) = U(\mathcal{R}) - U(\mathcal{S}) = \sum_{\theta\in\{0,1\}} \left[u(\theta,\theta)\Pr(d=\theta|\mathcal{R}) + u(1-\theta,\theta)\Pr(d\neq\theta|\mathcal{R})\right]$$

$$- \sum_{\theta\in\{0,1\}} \left[u(\theta,\theta)\Pr(d=\theta|\mathcal{S}) + u(1-\theta,\theta)\Pr(d\neq\theta|\mathcal{S})\right]$$

$$= \sum_{\theta\in\{0,1\}} u(\theta,\theta)\left[\Pr(d=\theta|\mathcal{R}) - \Pr(d=\theta|\mathcal{S})\right] + \sum_{\theta\in\{0,1\}} u(1-\theta,\theta)\left[\Pr(d\neq\theta|\mathcal{R}) - \Pr(d\neq\theta|\mathcal{S})\right]$$

$$= \sum_{\theta\in\{0,1\}} u(\theta,\theta)\left[\Pr(d=\theta|\mathcal{R}) - \Pr(d=\theta|\mathcal{S})\right] + \sum_{\theta\in\{0,1\}} u(1-\theta,\theta)\left[-\Pr(d=\theta|\mathcal{R}) + \Pr(d=\theta|\mathcal{S})\right]$$

$$= \sum_{\theta\in\{0,1\}} \left[u(\theta,\theta) - u(1-\theta,\theta)\right]\left[\Pr(d=\theta|\mathcal{R}) - \Pr(d=\theta|\mathcal{S})\right] = \Pr(d=\theta|\mathcal{R}) - \Pr(d=\theta|\mathcal{S}),$$

where the last equality uses $u(1,1) - u(0,1) = u(0,0) - u(1,0) = 1$.

Next, let T denote the vote tally among the remaining $N-1$ shareholders. The shareholder's vote only changes the decision d if the votes of other shareholders are split, and hence $\Pr(d=\theta|T,\mathcal{R}) - \Pr(d=\theta|T,\mathcal{S}) = 0$ if $T \neq \frac{N-1}{2}$. Hence,

$$W(\mathcal{R},\mathcal{S}) = \sum_{T=0}^{N-1} \Pr(T)\left[\Pr(d=\theta|T,\mathcal{R}) - \Pr(d=\theta|T,\mathcal{S})\right]$$

$$= \Pr\left(T=\frac{N-1}{2}\right)\left[\Pr\left(d=\theta|T=\frac{N-1}{2},\mathcal{R}\right) - \Pr\left(d=\theta|T=\frac{N-1}{2},\mathcal{S}\right)\right],$$

which gives (77), as required. Expression (77) follows from Bayes rule.

Special case when the research report is fully informative. If the research report is fully informative, then $\Pr(d=\theta|Piv,\mathcal{R}) = 1$, and

$$W(\mathcal{R},\mathcal{S}) = \sum_{s\in S} \Pr(s)\Pr(Piv|s)\left[1 - \Pr(d=\theta|Piv,s)\right].$$

We can split the set S into two subsets depending on the posterior μ_s. If μ_s is 0 or 1, then all shareholders know the state with certainty, and given the focus on undominated strategies, they all vote according to the state, so $\Pr(Piv|s) = 0$. If $\mu_s \in (0,1)$, then, as shown in the proof of Proposition 2, $\Pr(d=\theta|Piv,s) = \frac{1}{2}$. Hence,

$$W(\mathcal{R},\mathcal{S}) = \sum_{s\in S} \Pr(s)\Pr(Piv|s)\frac{1}{2},$$

i.e., we get expression (7), as required.

B Implications for empirical voting patterns

In this section, we present additional derivations for the empirical implications in Section 5. In Sections B.1 and B.2, we derive the expressions used in the numerical example in Figure 4. In Section B.3, we prove an additional statement used in Section 5.2.

B.1 Probability of a close vote

We focus on the case of $q^* < \frac{1}{2}$ and $\mu \geq \mu_0(q^*) = (1+(1-4q^{*2})^{\frac{N-1}{2}})^{-1}$. The case of $\mu \leq \mu_1(q^*)$ is similar by symmetry. Denote T the voting tally, i.e., the random variable that stands for the number of votes in favor of the proposal.

We first provide the comparison for the probability of a close vote, defined as:

$$\Pr(\textit{close vote}) \equiv \Pr\left(T = \frac{N-1}{2}\right) + \Pr\left(T = \frac{N+1}{2}\right).$$

As we show next, the probability of a close vote defined this way is proportional to the probability of a shareholder being pivotal (i.e., the probability of a split vote among $N-1$ shareholders). Hence, this probability is maximized under the optimal recommendation design $\mathcal{S}^*$ and is higher than under uninformative recommendations. Formally, if the recommendation takes the form described in Part 2 of Proposition 6, then a close vote never occurs upon $s = 1$, and hence

$$\Pr(\textit{close vote}|\mathcal{S}^*) = \Pr(\textit{close vote}|s=0)\Pr(s=0),$$

where

$$\Pr(\textit{close vote}|s=0) = \Pr(\textit{close vote}|\theta=1, s=0)\,\mu_0(q^*) + \Pr(\textit{close vote}|\theta=0, s=0)\,(1-\mu_0(q^*)).$$

Note that

$$\Pr(\textit{close vote}|\theta=1, s=0) = C_N^{\frac{N-1}{2}}\left[\varrho_1^*\right]^{\frac{N-1}{2}}\left[1-\varrho_1^*\right]^{\frac{N+1}{2}} + C_N^{\frac{N+1}{2}}\left[\varrho_1^*\right]^{\frac{N+1}{2}}\left[1-\varrho_1^*\right]^{\frac{N-1}{2}},$$

where using (23)-(24)

$$\varrho_1^* \equiv \varrho_1(q, \mu_0(q^*)) = q + (1-q)\,\pi(q, \mu_0(q^*)), \tag{79}$$

$$\varrho_0^* \equiv \varrho_0(q, \mu_0(q^*)) = (1-q)\,\pi(q, \mu_0(q^*)), \tag{80}$$

and $\pi(q, \cdot)$ is given by (4). Using $C_N^{\frac{N-1}{2}} = C_N^{\frac{N+1}{2}}$, we get

$$\Pr(\textit{close vote}|\theta=1, s=0) = C_N^{\frac{N-1}{2}}\left[\varrho_1^*\right]^{\frac{N-1}{2}}\left[1-\varrho_1^*\right]^{\frac{N-1}{2}}$$

$$= C_{N-1}^{\frac{N-1}{2}}\left[\varrho_1^*\right]^{\frac{N-1}{2}}\left[1-\varrho_1^*\right]^{\frac{N-1}{2}}\left(C_N^{\frac{N-1}{2}}/C_{N-1}^{\frac{N-1}{2}}\right) = \Pr(Piv|\theta=1, s=0)\,\frac{2N}{N+1},$$

where $\Pr(Piv|\cdot)$ is the probability of a shareholder being pivotal, i.e., the probability of a split vote among $N-1$ shareholders. Using the same argument for $\Pr(\textit{close vote}|\theta=0, s=0)$, we can rewrite

$$\Pr\left(close\ vote|\mathcal{S}^*\right) = \Pr\left(Piv|s=0\right)\Pr\left(s=0\right)\frac{2N}{N+1} = P\left(q^*,\mu\right)\frac{2N}{N+1},$$

where $P\left(q^*,\mu\right)$ is the average probability of a shareholder being pivotal under recommendation policy $\mathcal{S}^*$. By exactly the same arguments, for an uninformative recommendation, $\Pr\left(close\ vote|\mathcal{S}_{uninf}\right) = \Pr\left(Piv|q^*,\mu\right)\frac{2N}{N+1}$, where $\Pr\left(Piv|q^*,\mu\right)$ is given by (22). Since $P\left(q^*,\mu\right)$ is the maximum possible average probability of a shareholder being pivotal, we automatically have $\Pr\left(close\ vote|\mathcal{S}^*\right) > \Pr\left(close\ vote|\mathcal{S}_{uninf}\right)$. The two coincide when $\mu = \mu_0\left(q^*\right)$.

B.2 Probability of a lopsided vote

We next derive the probability of a lopsided vote, defined as:

$$\Pr\left(lopsided\ vote\right) \equiv \Pr\left(T=0\right) + \Pr\left(T=N\right).$$

For the optimal recommendation design $\mathcal{S}^*$, a lopsided vote occurs with probability one upon $s=1$, and hence

$$\Pr\left(lopsided\ vote|\mathcal{S}^*\right) = \frac{\mu-\mu_0\left(q^*\right)}{1-\mu_0\left(q^*\right)} + \Pr\left(lopsided\ vote|s=0\right)\frac{1-\mu}{1-\mu_0\left(q^*\right)},$$

where

$$\begin{aligned}\Pr\left(lopsided\ vote|s=0\right) &= \Pr\left(lopsided\ vote|\theta=1,s=0\right)\mu_0\left(q^*\right) \\ &\quad + \Pr\left(lopsided\ vote|\theta=0,s=0\right)\left(1-\mu_0\left(q^*\right)\right) \\ &= \left(\left[\varrho_1^*\right]^N + \left[1-\varrho_1^*\right]^N\right)\mu_0\left(q^*\right) + \left(\left[\varrho_0^*\right]^N + \left[1-\varrho_0^*\right]^N\right)\left(1-\mu_0\left(q^*\right)\right),\end{aligned}$$

and ϱ_1^*, ϱ_0^* are given by (79)-(80). Similary, for an uninformative recommendation,

$$\begin{aligned}\Pr\left(lopsided\ vote|\mathcal{S}_{uninf}\right) &= \Pr\left(lopsided\ vote|\theta=1\right)\mu + \Pr\left(lopsided\ vote|\theta=0\right)\left(1-\mu\right) \\ &= \left(\left[\varrho_1\right]^N + \left[1-\varrho_1\right]^N\right)\mu + \left(\left[\varrho_0\right]^N + \left[1-\varrho_0\right]^N\right)\left(1-\mu\right),\end{aligned}$$

and using (23)-(24)

$$\varrho_1 \equiv \varrho_1\left(q,\mu\right) = q + \left(1-q\right)\pi\left(q,\mu\right), \tag{81}$$

$$\varrho_0 \equiv \varrho_0\left(q,\mu\right) = \left(1-q\right)\pi\left(q,\mu\right), \tag{82}$$

and $\pi\left(q,\cdot\right)$ is given by (4).

For any N, as $q \to 0$, we have $\varrho_1 \to 1$ and $\varrho_0 \to 1$, so $\Pr\left(lopsided\ vote|\mathcal{S}_{uninf}\right) \to 1$. For the optimal recommendation $\mathcal{S}^*$, $\lim_{q\to 0}\mu_0\left(q\right) \to \frac{1}{2}$, so $\pi\left(q,\mu_0\left(q^*\right)\right) \to \frac{1}{2}$, and hence $\varrho_1^* \to \frac{1}{2}$, $\varrho_0^* \to \frac{1}{2}$, so $\Pr\left(lopsided\ vote|\mathcal{S}^*\right) \to \left(2\mu-1\right)\left(1-2^{1-N}\right)$, which is bounded away from 1.

B.3 Effect of the fraction of informed shareholders on recommendation design

We next prove an additional statement used in the empirical implications of Section 5.2. Let q be the expected fraction of shareholders who observe the state, where q could correspond to q^* (the equilibrium expected fraction of subscribers) in the main model or to q_e^* (the equilibrium expected fraction of subscribers plus the fraction χ of exogenously informed shareholders) in the extension of Section 4.2. We prove the statement used in Implication 5, that the probability of a close vote conditional on the "controversial" recommendation decreases in q.

To prove this, consider the case of $\mu \leq \mu_1(q)$, when the "controversial" recommendation is $s = 1$ (the case of $\mu \geq \mu_0(q)$ is analogous). We first show that the probability of a shareholder being pivotal conditional on $s = 1$ decreases in q. Indeed, according to the proof of Proposition 4, the average probability of a shareholder being pivotal given the optimal recommendation design is $\mu C_{N-1}^{\frac{N-1}{2}} 2^{2-N}$. Since a shareholder is never pivotal upon $s = 0$ and since $\Pr(s = 1) = \frac{\mu}{\mu_1(q)}$, it follows that

$$\Pr(Piv|s = 1) \frac{\mu}{\mu_1(q)} = \mu C_{N-1}^{\frac{N-1}{2}} 2^{2-N}.$$

Since $\mu_1(q)$ decreases in q, $\Pr(Piv|s = 1)$ decreases in q as well. Finally, as shown in Section B.1 of the Online Appendix above, the probability of a close vote (i.e., a vote tally of $\frac{N-1}{2}$ or $\frac{N+1}{2}$) is $\frac{2N}{N+1}$ times the probability of a shareholder being pivotal, and hence the probability of a close vote conditional on $s = 1$ also decreases in q.

References

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