Online Appendix for "The Voting Premium"

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A Supplemental Analysis

Lemma 3. *Let* $\underline{v} \equiv v_0 + \min\left\{-\overline{b}, \beta\right\}$ *and* $\overline{v} \equiv v_0 + \max\left\{\overline{b}, \beta\right\} + 1$. *Then,*

(i) *If* $\gamma > \frac{\overline{v}-\underline{v}}{1-\alpha}$ *then no dispersed shareholder short-sells the share in any equilibrium, that is,* $x^*(b) + 1 - \alpha > 0$ *for any* $b \in \left[-\overline{b}, \overline{b}\right]$.

(ii) *Suppose* $\alpha > 0$. *If* $\eta > \frac{4(\overline{v}-\underline{v})}{\alpha}$ *then the blockholder never short-sells the share in any equilibrium, that is,* $y^* + \alpha > 0$.

(iii) *If* $\alpha < \min\{\tau, 1-\tau\}$ *and* $\eta > \frac{2(\overline{v}-\underline{v})\min\{\tau,1-\tau\}}{(\min\{\tau,1-\tau\}-\alpha)^2}$ *then the blockholder never obtains a control stake or veto power in equilibrium, that is,* $\alpha + y^* < \min\{\tau, 1-\tau\}$.

Proof. The valuation of the share by any investor is bounded from above by $\overline{v}$ and from below by $\underline{v}$. Therefore, in any equilibrium, $|p^* - v(b, q^*)| < \overline{v} - \underline{v}$ and $|p^* - v(\beta, q^*)| < \overline{v} - \underline{v}$. Based on (7), $x(b, p^*) + 1 - \alpha > 0 \Leftrightarrow \gamma > \frac{p^* - v(b,q^*)}{1-\alpha}$. Therefore, requiring $\gamma > \frac{\overline{v}-\underline{v}}{1-\alpha}$ guarantees that in any equilibrium $x(b, p^*) + 1 - \alpha > 0$ as required by part (i).

Consider the blockholder. First, if the blockholder chooses $y^* < 0$ in equilibrium then it must be

$$
\begin{aligned}
\Pi(y^*) &\geq \Pi(0) \Leftrightarrow \\
y^*\left[v(\beta, q^*(y^*)) - p^*(y^*)\right] - \frac{\eta}{2}y^{*2} &\geq \alpha\left[v(\beta, q^*(0)) - v(\beta, q^*(y^*))\right] \Rightarrow \\
-y^*(\overline{v} - \underline{v}) - \frac{\eta}{2}y^{*2} &\geq -\alpha(\overline{v} - \underline{v}) \Leftrightarrow \\
y^* &\geq -\frac{(\overline{v} - \underline{v}) + \sqrt{(\overline{v} - \underline{v})^2 + 2\alpha\eta(\overline{v} - \underline{v})}}{\eta}.
\end{aligned}
$$

Therefore, assuming

$$
-\frac{(\overline{v} - \underline{v}) + \sqrt{(\overline{v} - \underline{v})^2 + 2\alpha\eta(\overline{v} - \underline{v})}}{\eta} > -\alpha \Leftrightarrow \eta > \frac{4(\overline{v} - \underline{v})}{\alpha}
$$

guarantees that the blockholder will sell less than his entire endowment in any equilibrium, that is, $y^* > -\alpha$ as required by part (ii). Second, if the blockholder chooses $y^* > 0$ in equilibrium

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then it must be

$$\begin{aligned}
\Pi(y^*) &\geq \Pi(0) \Leftrightarrow \\
y^*\left[v(\beta, q^*(y^*)) - p^*(y^*)\right] - \frac{\eta}{2} y^{*2} &\geq \alpha\left[v(\beta, q^*(0)) - v(\beta, q^*(y^*))\right] \Rightarrow \\
y^*(\overline{v} - \underline{v}) - \frac{\eta}{2} y^{*2} &\geq -\alpha(\overline{v} - \underline{v}) \Leftrightarrow \\
y^* &\leq \frac{\overline{v} - \underline{v} + \sqrt{(\overline{v} - \underline{v})^2 + 2\alpha\eta(\overline{v} - \underline{v})}}{\eta}.
\end{aligned}$$

Therefore, assuming

$$\frac{\overline{v} - \underline{v} + \sqrt{(\overline{v} - \underline{v})^2 + 2\alpha\eta(\overline{v} - \underline{v})}}{\eta} < \min\{\tau, 1-\tau\} - \alpha \Leftrightarrow \eta > \frac{2(\overline{v} - \underline{v})\min\{\tau, 1-\tau\}}{(\min\{\tau, 1-\tau\} - \alpha)^2}$$

guarantees that the blockholder will not obtain a stake larger than $\min\{\tau, 1-\tau\}$ in any equilibrium, that is, $y^* + \alpha < \min\{\tau, 1-\tau\}$ as required by part (iii). ■

B Voting yields and capitalized voting premiums

The dual-class share premium is commonly computed as the relative price difference between voting and non-voting shares. Let $P_{t,v}$ be the price per voting share and $P_{t,nv}$ the price per non-voting shares in period t:[33]

$$\text{Dual class premium}_t = \frac{P_{t,v} - P_{t,nv}}{P_{t,v}}. \tag{75}$$

The dual-class premium captures the potentially infinite time horizon over which the owner of a block of voting rights enjoys control rights, which ends only if the firm ceases to exist, e.g., because of acquisitions or insolvencies, or when the two classes of shares are unified into one class. Hence, they represent the capitalized value of a the right to vote at all future shareholder meetings. The same is true for the block-trading premium, which is discussed below.

By contrast, the last three methods in our list measure the value of voting rights only for very limited periods of time ranging from three days to 57 days.[34] These time spans do not capture more than one shareholder meeting. Hence, these methods estimate a voting yield,

[33] This statistic applies only if one class of shares has no voting rights and both classes have the same par value. It is appropriately adjusted when par values differ (Megginson (1990)) or when computing the value of control for firms that have two classes of voting shares, but different ratios of cash flow rights to voting rights; see, e.g., Zingales (1995). Bigelli and Croci (2013) Argue that many studies lack appropriate adjustments for differential dividends.

[34] Table 1 reports annualized figures if such figures are reported by the authors. Kalay, Karakas, and Pant (2014) and Gurun and Karakas (2020) construct non-voting shares synthetically from options with an average maturity of, respectively, 38 days and 57 days. the equity-lending method (Christoffersen et al. (2007); Aggarwal, Saffi, and Sturgess (2015)) investigates fees for lending shares around record dates, and the record-day trading method measures stock price drops in a 3-day trading window around record dates (Fos and Holderness (2020)).

which has the same dimension as a dividend yield. Let V_t represent the per-share dollar value of a voting right and D_t the dollar value of dividends per share, where the subscript indexes time. Then $D_t/P_{t-1,v}$ and $D_t/P_{t-1,nv}$ are the dividend yields of, respectively, the voting and the non-voting shares, and $V_t/P_{t-1,nv}$ is the voting yield. Let r_v be the constant per-period discount rate for the voting shares and r_{nv} the constant per-period discount rate for the non-voting shares, and assume the value of voting rights and dividends both grow at the same constant rate g, which allows us to calculate the value of voting shares and of non-voting shares using the Gordon growth formula:

$$P_{0,v} = \frac{D_1 + V_1}{r_v - g}; \; P_{0,nv} = \frac{D_1}{r_{nv} - g}. \tag{76}$$

Solving both expressions for g, equating them, and rearranging gives:

$$\frac{V_1}{P_{0,v}} = \frac{P_{0,v} - P_{0,nv}}{P_{0,nv}} \frac{D_1}{P_{0,v}} + r_v - r_{nv}. \tag{77}$$

Note, however, that numerous aspects are missing in this simple analysis, e.g., the time horizon for control may be finite because of possible mergers, stock unifications, or regulatory change. See, e.g., Goetzmann, Ukhov, and Spiegel (2002).If we assume that $r_v = r_{nv}$, we obtain:

$$\text{Voting yield} = \text{ Dual-class premium } \times \text{ Dividend yield.}$$

This relationship is broadly consistent with US data. A dual-class share premium of about 5%-10% (see Table 1), multiplied by a dividend yield of 3.7% (see Fama and French (2002) for the period 1951-2000) results in estimates of the voting yield of 0.18% to 0.37%, which compares well to estimates of the annualized voting yield from option replications (Kalay, Karakas, and Pant (2014): 0.16%; Kind and Poltera (2013): 0.37%). If we relax the assumption that $r_v = r_{nv}$, then from (77) we would expect a lower voting yield if the discount rate for voting shares is higher than that for non-voting shares, i.e., if investors consider votes to be riskier than dividends.

C Analysis of multiple blockholders

C.1 Multiple homogenous blockholders

Assume there are $N \geq 2$ blockholders, each blockholder is endowed with $\alpha_i = \frac{\alpha}{N}$ shares. All blockholders face the same trading cost η and have the same bias β.

Proposition 8. *There exist $\overline{\gamma} < \infty$ and $\overline{\eta} < \infty$ such that if $\gamma > \overline{\gamma}$ and $\eta > \overline{\eta}$, then the equilibrium exists and is unique. The equilibrium trade of each blockholder, denoted by y^*, satisfies*

$$Ny^* = \frac{N}{\gamma (N+1) + \eta} (\beta - \mathbb{E}[b]) H (q^* (Ny^*)) + \frac{1}{\gamma (N+1) + \eta} MPV (Ny^*), \tag{78}$$

and the share price is given by

$$p^*(N) = v(b_{MT}(N), q^*(Ny^*)) + \frac{\gamma}{\gamma(N+1)+\eta} MPV(Ny^*),$$

where $q^*(\cdot)$ is given in Proposition 1 and

$$b_{MT}(N) = \frac{\gamma+\eta}{\gamma(N+1)+\eta}\mathbb{E}[b] + \frac{\gamma N}{\gamma(N+1)+\eta}\beta. \tag{79}$$

Proof of Proposition 8. We denote the trade of blockholder $i \in \{1, ..., N\}$ by y_i. Let $y = \sum_{i=1}^N y_i$ and $y_{-i} = \sum_{j \neq i} y_j$. Since all blockholders have the same bias, Proposition 1 holds with respect to y. Moreover, given y and $q^*(y)$, trade by dispersed shareholders is also the same as in the baseline model, and in particular, the share price is given by $p^*(y) = \gamma y + v(\mathbb{E}[b], q^*(y))$. The profit of blockholder i is given by

$$\begin{aligned}
\Pi(y_i, y_{-i}) &= (\alpha_i + y_i) v(\beta, q^*(y_{-i}+y_i)) - y_i p^*(y_{-i}+y_i) - \frac{\eta}{2}(y_i)^2 \\
&= \alpha_i v(\beta, q^*(y_{-i}+y_i)) + y_i(\beta - \mathbb{E}[b]) H(q^*(y_{-i}+y_i)) - y_i\gamma y_{-i} - (\gamma+\eta/2)(y_i)^2 \\
&= \alpha_i v_0 + \alpha_i \mathbb{E}[\theta | q > q^*(y_{-i}+y_i)] H(q^*(y_{-i}+y_i)) \\
&\quad ((\alpha_i + y_i)\beta - y_i \mathbb{E}[b]) H(q^*(y_{-i}+y_i)) - (\gamma+\eta/2)(y_i)^2 - y_i \gamma y_{-i}.
\end{aligned}$$

The derivative of $\Pi(y_i, y_{-i})$ with respect to y_i, $\Pi'(y_i, y_{-i})$, is given by

$$\begin{aligned}
\Pi'(y_i, y_{-i}) &= \underbrace{(\beta - \mathbb{E}[b]) H(q^*(y_{-i}+y_i)) - (2\gamma+\eta) y_i - \gamma y_{-i}}_{\text{marginal propensity to buy cash flows}} \\
&\quad + \underbrace{\frac{\partial(-q^*(y_{-i}+y_i))}{\partial y} f(q^*(y_{-i}+y_i)) [\alpha_i (q^*(y_{-i}+y_i)+\beta) + y_i(\beta - \mathbb{E}[b])]}_{\text{marginal propensity to buy votes}}.
\end{aligned}$$

The symmetry across blockholders requires all of them to trade the same amount, and thus, the equilibrium level of y_i^* satisfies

$$\begin{aligned}
0 &= \begin{bmatrix} (\beta - \mathbb{E}[b]) H(q^*(Ny_i^*)) - (2\gamma+\eta) y_i^* - \gamma(N-1) y_i^* \\ + \frac{\partial(-q^*(Ny_i^*))}{\partial y} f(q^*(Ny_i^*)) \left[\frac{\alpha}{N}(q^*(Ny_i^*)+\beta) + y_i^*(\beta - \mathbb{E}[b])\right] \end{bmatrix} \Leftrightarrow \\
Ny_i^* &= \frac{N}{\gamma(N+1)+\eta}(\beta - \mathbb{E}[b]) H(q^*(Ny_i^*)) + \frac{1}{\gamma(N+1)+\eta} MPV(Ny_i^*).
\end{aligned}$$

The share price in equilibrium is

$$\begin{aligned} p^*(N) &= \gamma N y_i^* + v\left(\mathbb{E}[b], q^*(Ny_i^*)\right) \\ &= \gamma\left[\frac{N}{\gamma(N+1)+\eta}(\beta-\mathbb{E}[b])H(q^*(Ny_i^*)) + \frac{1}{\gamma(N+1)+\eta}MPV(Ny_i^*)\right] \\ &\quad + v\left(\mathbb{E}[b], q^*(Ny_i^*)\right) \\ &= v_0 + \left(\frac{\gamma N}{\gamma(N+1)+\eta}(\beta-\mathbb{E}[b]) + \mathbb{E}[b]\right)H(q^*(Ny_i^*)) \\ &\quad + \mathbb{E}[\theta|q > q^*(Ny_i^*)]H(q^*(Ny_i^*)) + \frac{\gamma}{\gamma(N+1)+\eta}MPV(Ny_i^*) \\ &= v\left(\frac{\gamma+\eta}{\gamma(N+1)+\eta}\mathbb{E}[b] + \frac{\gamma N}{\gamma(N+1)+\eta}\beta, q^*(Ny_i^*)\right) + \frac{\gamma}{\gamma(N+1)+\eta}MPV(Ny_i^*) \\ &= v\left(b_{MT}(N), q^*(Ny_i^*)\right) + \frac{\gamma}{\gamma(N+1)+\eta}MPV(Ny_i^*), \end{aligned}$$

as required. ■

Hence, in the limit, the price is the valuation of the blockholders absent any voting premium, that is, $\lim_{N\to\infty} p^*(N) = v(\beta, q^*(y_\infty^*))$, where y_∞^* satisfies $y_\infty^* = \frac{1}{\gamma}(\beta - \mathbb{E}[b])H(q^*(y_\infty^*))$.

C.2 Heterogenous blockholders

Suppose μN blockholders have bias $\beta_c \approx -\overline{b}$, and $(1-\mu)N$ blockholders have bias $\beta_a \approx \overline{b}$, where $\mu \in (0,1)$ is such that both μN and $(1-\mu)N$ are integers. Thus, blockholders with bias β_c always (i.e., regardless of the realization of signal q) vote against the proposal, and blockholders with bias β_a always vote for the proposal. We let $\overline{\beta} = (1-\mu)\beta_a + \mu\beta_c$ and $q^*(y_c, y_a)$ be the solution of

$$s(-q^*; y_c + y_a, q^*) + (1-\mu)\alpha + y_a = \tau. \tag{80}$$

The following result holds.

Proposition 9. *There exist $\overline{\gamma} < \infty$ and $\overline{\eta} < \infty$ such that if $\gamma > \overline{\gamma}$ and $\eta > \overline{\eta}$, then the equilibrium exists and is unique. In equilibrium, blockholders biased in favor (against) the proposal trade y_a^* (y_c^*) shares such that*

$$\begin{aligned} N(\mu y_c^* + (1-\mu)y_a^*) &= \frac{N}{\gamma(N+1)+\eta}\left(\overline{\beta} - \mathbb{E}[b]\right)H(q^{**}) \\ &\quad + \frac{1}{\gamma(N+1)+\eta}\left(\mu MPV_c^{**} + (1-\mu)MPV_a^{**}\right), \end{aligned} \tag{81}$$

and the share price is given by

$$p^*(N) = v\left(b_{MT}(N,\mu), q^{**}\right) + \frac{\gamma}{\gamma(N+1)+\eta}\left(\mu MPV_c^{**} + (1-\mu)MPV_a^{**}\right), \tag{82}$$

where

$$\begin{aligned}
q^{**} &= q^*\left(\mu N y_c^*, (1-\mu) N y_a^*\right), \\
b_{MT}(N) &= \frac{\gamma+\eta}{\gamma(N+1)+\eta}\mathbb{E}[b] + \frac{\gamma N}{\gamma(N+1)+\eta}\overline{\beta}, \\
MPV_a^{**} &= \frac{\partial\left(-q^*(y_c, y_a)\right)}{\partial y_a}|_{q^*=q^{**}} f(q^{**})\left[\alpha\left(q^{**}+\beta_a\right) + N y_a^*\left(\beta_a - \mathbb{E}[b]\right)\right], \\
MPV_c^{**} &= \frac{\partial\left(-q^*(y_c, y_a)\right)}{\partial y_c}|_{q^*=q^{**}} f(q^{**})\left[\alpha\left(q^{**}+\beta_c\right) + N y_c^*\left(\beta_c - \mathbb{E}[b]\right)\right].
\end{aligned}$$

Moreover, as blockholders become more extreme in the sense that β_a increases by ε and β_c decreases by ε, the aggregate MPV, $\mu MPV_c^{**} + (1-\mu) MPV_a^{**}$, increases.

Proof of Proposition 9. We denote the aggregate trade of all blockholders biased against (in favor) of the proposal by y_c (y_a). Given $y = y_c + y_a$, the trade of dispersed investors is as in the baseline model. Given q and the expectations of dispersed shareholders that the proposal will be approved if and only if $q > q_e^*$, the number of affirmative votes by dispersed investors is $s(q; y, q_e^*)$. Thus, the proposal is accepted if and only if

$$s(q; y, q_e^*) + (1-\mu)\alpha + y_a \geq \tau. \tag{83}$$

Thus, in equilibrium, the marginal voter is a dispersed investor whose bias $-q^*$ solves

$$\begin{gathered}
s(-q^*; y, q^*) + (1-\mu)\alpha + y_a = \tau \Leftrightarrow \\
(1-\alpha-y_c-y_a)\left[1 - G(-q^*)\left(1 - \frac{\mathbb{E}[b]-\mathbb{E}[b|b<-q^*]}{\gamma}\frac{H(q^*)}{1-\alpha-y_c-y_a}\right)\right] + (1-\mu)\alpha + y_a = \tau \Leftrightarrow \\
1 - G(-q^*)\left(1 - \frac{\mathbb{E}[b]-\mathbb{E}[b|b<-q^*]}{\gamma}\frac{H(q^*)}{1-\alpha-y_c-y_a}\right) = \frac{\tau-(1-\mu)\alpha-y_a}{1-\alpha-y_c-y_a}.
\end{gathered}$$

If γ is sufficiently high, the solution is unique, as in the baseline model. Suppose this is the case. Then,

$$\frac{\partial(-q^*)}{\partial y_c} = -\frac{G(-q^*)\left(\frac{\mathbb{E}[b]-\mathbb{E}[b|b<-q^*]}{\gamma}\frac{H(q^*)}{(1-\alpha-y_c-y_a)^2}\right) - \frac{\tau-(1-\mu)\alpha-y_a}{(1-\alpha-y_c-y_a)^2}}{\frac{\partial\left(1-G(-q^*)\left(1-\frac{\mathbb{E}[b]-\mathbb{E}[b|b<-q^*]}{\gamma}\frac{H(q^*)}{1-\alpha-y_c-y_a}\right)\right)}{\partial(-q^*)}}.$$

Notice that the denominator is negative (since the solution is unique for a large γ) and that the numerator can be written as

$$\begin{aligned}
& \frac{1}{1-\alpha-y_c-y_a}\left[G(-q^*)\frac{\mathbb{E}[b]-\mathbb{E}[b|b<-q^*]}{\gamma}\frac{H(q^*)}{1-\alpha-y_c-y_a} - \frac{\tau-(1-\mu)\alpha-y_a}{1-\alpha-y_c-y_a}\right] \\
= & \frac{1}{1-\alpha-y_c-y_a}\left[\begin{array}{c} G(-q^*)\frac{\mathbb{E}[b]-\mathbb{E}[b|b<-q^*]}{\gamma}\frac{H(q^*)}{1-\alpha-y_c-y_a} \\ -\left(1-G(-q^*)\left(1-\frac{\mathbb{E}[b]-\mathbb{E}[b|b<-q^*]}{\gamma}\frac{H(q^*)}{1-\alpha-y_c-y_a}\right)\right)\end{array}\right] \\
= & \frac{1}{1-\alpha-y_c-y_a}\left[-1+G(-q^*)\right] < 0,
\end{aligned}$$

so $\frac{\partial(-q^*)}{\partial y_c} < 0$. That is, the marginal voter becomes more biased against the proposal as blockholders biased against the proposal buy more shares. Next, notice that

$$\begin{aligned}
\frac{\partial\left(-q^*\right)}{\partial y_a} &= -\frac{\frac{1}{1-\alpha-y_c-y_a}\left[G\left(-q^*\right)\frac{\mathbb{E}[b]-\mathbb{E}[b|b<-q^*]}{\gamma}\frac{H(q^*)}{1-\alpha-y_c-y_a}-\frac{\tau-1+\mu\alpha+y_c}{1-\alpha-y_c-y_a}\right]}{\frac{\partial\left(1-G(-q^*)\left(1-\frac{\mathbb{E}[b]-\mathbb{E}[b|b<-q^*]}{\gamma}\frac{H(q^*)}{1-\alpha-y_c-y_a}\right)\right)}{\partial(-q^*)}} \\
&= -\frac{\frac{1}{1-\alpha-y_c-y_a}\left[\frac{\tau-(1-\mu)\alpha-y_a}{1-\alpha-y_c-y_a}-\left(1-G\left(-q^*\right)\right)-\frac{\tau-1+\mu\alpha+y_c}{1-\alpha-y_c-y_a}\right]}{\frac{\partial\left(1-G(-q^*)\left(1-\frac{\mathbb{E}[b]-\mathbb{E}[b|b<-q^*]}{\gamma}\frac{H(q^*)}{1-\alpha-y_c-y_a}\right)\right)}{\partial(-q^*)}} \\
&= -\frac{\frac{1}{1-\alpha-y_c-y_a}G\left(-q^*\right)}{\frac{\partial\left(1-G(-q^*)\left(1-\frac{\mathbb{E}[b]-\mathbb{E}[b|b<-q^*]}{\gamma}\frac{H(q^*)}{1-\alpha-y_c-y_a}\right)\right)}{\partial(-q^*)}} > 0,
\end{aligned}$$

so $\frac{\partial(-q^*)}{\partial y_a} > 0$. That is, the marginal voter becomes more activist as the activist blockholders buy more shares. Notice that

$$\begin{aligned}
\frac{\partial\left(-q^*\right)}{\partial y_c} &= -\frac{\frac{1}{1-\alpha-y_c-y_a}G\left(-q^*\right)}{\frac{\partial\left(1-G(-q^*)\left(1-\frac{\mathbb{E}[b]-\mathbb{E}[b|b<-q^*]}{\gamma}\frac{H(q^*)}{1-\alpha-y_c-y_a}\right)\right)}{\partial(-q^*)}} \\
&= -\frac{G\left(-q^*\right)}{-1+G\left(-q^*\right)}\frac{\frac{1}{1-\alpha-y_c-y_a}\left[-1+G\left(-q^*\right)\right]}{\frac{\partial\left(1-G(-q^*)\left(1-\frac{\mathbb{E}[b]-\mathbb{E}[b|b<-q^*]}{\gamma}\frac{H(q^*)}{1-\alpha-y_c-y_a}\right)\right)}{\partial(-q^*)}} \\
&= -\frac{G\left(-q^*\right)}{1-G\left(-q^*\right)}\frac{\partial\left(-q^*\right)}{\partial y_a}.
\end{aligned}$$

The profit of an activist blockholder i is given by

$$\begin{aligned}
\Pi_a\left(y_i, y_c, y_{a,-i}\right) &= \left(\alpha_i+y_i\right)v\left(\beta_a, q^*\left(y_c, y_{a,-i}+y_i\right)\right)-y_i p^*\left(y_c+y_{a,-i}+y_i\right)-\frac{\eta}{2}\left(y_i\right)^2 \\
&= \alpha_i v\left(\beta_a, q^*\left(y_c, y_{a,-i}+y_i\right)\right)+y_i\left(\beta_a-\mathbb{E}\left[b\right]\right)H\left(q^*\left(y_c, y_{a,-i}+y_i\right)\right) \\
&\quad -y_i\gamma\left(y_c+y_{a,-i}\right)-\left(\gamma+\eta/2\right)\left(y_i\right)^2 \\
&= \alpha_i v_0+\alpha_i\mathbb{E}\left[\theta|q>q^*\left(y_c, y_{a,-i}+y_i\right)\right]H\left(q^*\left(y_c, y_{a,-i}+y_i\right)\right) \\
&\quad \left(\left(\alpha_i+y_i\right)\beta_a-y_i\mathbb{E}\left[b\right]\right)H\left(q^*\left(y_c, y_{a,-i}+y_i\right)\right)-\left(\gamma+\eta/2\right)\left(y_i\right)^2-y_i\gamma\left(y_c+y_{a,-i}\right),
\end{aligned}$$

and thus, the first order condition implies

$$\left[\begin{array}{c}\left(\beta_a-\mathbb{E}\left[b\right]\right)H\left(q^*\left(y_c, y_{a,-i}+y_i\right)\right)-\left(2\gamma+\eta\right)y_i-\gamma\left(y_c+y_{a,-i}\right)+ \\ \frac{\partial(-q^*(y_c,y_{a,-i}+y_i))}{\partial y_a}f\left(q^*\left(y_c, y_{a,-i}+y_i\right)\right)\left[\alpha_i\left(q^*\left(y_c, y_{a,-i}+y_i\right)+\beta_a\right)+y_i\left(\beta_a-\mathbb{E}\left[b\right]\right)\right]\end{array}\right]=0.$$

Similarly, the profit of a blockholder i biased against the proposal is given by

$$\begin{aligned}
\Pi_c\left(y_i, y_{c,-i}, y_a\right) &= \alpha_i v_0+\alpha_i\mathbb{E}\left[\theta|q>q^*\left(y_{c,-i}+y_i, y_{a,}\right)\right]H\left(q^*\left(y_{c,-i}+y_i, y_{a,}\right)\right) \\
&\quad \left(\left(\alpha_i+y_i\right)\beta_c-y_i\mathbb{E}\left[b\right]\right)H\left(q^*\left(y_{c,-i}+y_i, y_{a,}\right)\right)-\left(\gamma+\eta/2\right)\left(y_i\right)^2-y_i\gamma\left(y_{c,-i}+y_a\right)
\end{aligned}$$

and thus, the first order condition implies

$$\left[\begin{array}{c}\left(\beta_c-\mathbb{E}[b]\right)H\left(q^*\left(y_{c,-i}+y_i,y_{a,}\right)\right)-(2\gamma+\eta)\,y_i-\gamma\left(y_{c,-i}+y_a\right)+\\ \frac{\partial\left(-q^*\left(y_{c,-i}+y_i,y_{a,}\right)\right)}{\partial y_c}f\left(q^*\left(y_{c,-i}+y_i,y_{a,}\right)\right)\left[\alpha_i\left(q^*\left(y_{c,-i}+y_i,y_{a,}\right)+\beta_c\right)+y_i\left(\beta_c-\mathbb{E}[b]\right)\right]\end{array}\right]=0.$$

The symmetry across blockholders with bias β_a implies that they all choose y_a^*, and the symmetry across blockholders with bias β_c implies that they all choose y_c^*. Let

$$q^{**}\equiv q^*\left(\mu N y_c^*,(1-\mu)\,N y_a^*\right).$$

Then, the two FOC conditions are reduced to

$$\begin{array}{c}\left(\beta_a-\mathbb{E}[b]\right)H\left(q^{**}\right)-(\gamma+\eta)\,y_a^*-\gamma\left(\mu y_c^*+(1-\mu)\,y_a^*\right)N\\ +\frac{\partial(-q^{**})}{\partial y_a}f\left(q^{**}\right)\left[\frac{\alpha}{N}\left(q^{**}+\beta_a\right)+y_a^*\left(\beta_a-\mathbb{E}[b]\right)\right]=0\end{array}$$

and

$$\begin{array}{c}\left(\beta_c-\mathbb{E}[b]\right)H\left(q^{**}\right)-(\gamma+\eta)\,y_c^*-\gamma\left(\mu y_c^*+(1-\mu)\,y_a^*\right)N\\ +\frac{\partial(-q^{**})}{\partial y_c}f\left(q^{**}\right)\left[\frac{\alpha}{N}\left(q^{**}+\beta_c\right)+y_c^*\left(\beta_c-\mathbb{E}[b]\right)\right]=0.\end{array}$$

The price is

$$p^*=\gamma\left(\mu N y_c^*,(1-\mu)\,N y_a^*\right)+v\left(\mathbb{E}[b]\,,q^*\left(\mu N y_c^*,(1-\mu)\,N y_a^*\right)\right).$$

Multiplying the FOC of blockholders with bias β_a by $(1-\mu)$ and the FOC of blockholders with bias β_c by μ, and adding the two outcomes, we get

$$\begin{aligned}&\left[\mu y_c^*+(1-\mu)\,y_a^*\right]N\\ =\;&\frac{N}{\gamma(N+1)+\eta}\left((1-\mu)\,\beta_a+\mu\beta_c-\mathbb{E}[b]\right)H\left(q^{**}\right)\\ &+\frac{1}{\gamma(N+1)+\eta}\left[\begin{array}{c}(1-\mu)\frac{\partial(-q^{**})}{\partial y_a}f\left(q^{**}\right)\left[\alpha\left(q^{**}+\beta_a\right)+N y_a^*\left(\beta_a-\mathbb{E}[b]\right)\right]\\ +\mu\frac{\partial(-q^{**})}{\partial y_c}f\left(q^{**}\right)\left[\alpha\left(q^{**}+\beta_c\right)+N y_c^*\left(\beta_c-\mathbb{E}[b]\right)\right]\end{array}\right]\\ =\;&\frac{N}{\gamma(N+1)+\eta}\left(\overline{\beta}-\mathbb{E}[b]\right)H\left(q^{**}\right)+\frac{1}{\gamma(N+1)+\eta}\left[\mu MPV_c^{**}+(1-\mu)\,MPV_a^{**}\right].\end{aligned}$$

Thus, we can write the share price as

$$p^*=v\left(b_{MT}(N)\,,q^{**}\right)+\frac{1}{\gamma(N+1)+\eta}\left[\mu MPV_c^{**}+(1-\mu)\,MPV_a^{**}\right].$$

Next, notice that

$$\begin{aligned}\lim_{\gamma\to\infty}\frac{\partial\left(-q^*\right)}{\partial y_a}&=\frac{1}{1-\alpha}\frac{G\left(-q^*\right)}{g\left(-q^*\right)}>0,\\ \lim_{\gamma\to\infty}\frac{\partial\left(-q^*\right)}{\partial y_c}&=-\frac{G\left(-q^*\right)}{1-G\left(-q^*\right)}\frac{1}{1-\alpha}\frac{G\left(-q^*\right)}{g\left(-q^*\right)}<0,\end{aligned}$$

and thus,

$$\begin{aligned}\lim_{\gamma\to\infty} MPV_a^{**} &= \frac{1}{1-\alpha}\frac{G\left(-q^{**}\right)}{g\left(-q^{**}\right)} f\left(q^{**}\right)\frac{\alpha}{N}\left(q^{**}+\beta_a\right) > 0,\\ \lim_{\gamma\to\infty} MPV_c^{**} &= -\frac{G\left(-q^{**}\right)}{1-G\left(-q^{**}\right)}\frac{1}{1-\alpha}\frac{G\left(-q^{**}\right)}{g\left(-q^{**}\right)} f\left(q^{**}\right)\frac{\alpha}{N}\left(q^{**}+\beta_c\right) > 0.\end{aligned}$$

Also notice that $\lim_{\gamma\to\infty} q^{**}$ solves

$$1-G\left(-q^{**}\right) = \frac{\tau-(1-\mu)\alpha}{1-\alpha}.$$

Thus,

$$\begin{aligned}\lim_{\gamma\to\infty}\left[\mu MPV_c^{**}+(1-\mu) MPV_a^{**}\right] &= \frac{1}{1-\alpha}\frac{G\left(-q^{**}\right)}{g\left(-q^{**}\right)} f\left(q^{**}\right)\frac{\alpha}{N}\\ &\quad\times\left[q^{**}+(1-\mu)\beta_a+\mu\beta_c-\mu\frac{q^{**}+\beta_c}{1-G\left(-q^{**}\right)}\right].\end{aligned}$$

Suppose β_a increases by $\varepsilon>0$ and β_c decreases by ε. Then, we can write

$$\begin{aligned}\lim_{\gamma\to\infty}\left[\mu MPV_c^{**}+(1-\mu) MPV_a^{**}\right] &= \frac{1}{1-\alpha}\frac{G\left(-q^{**}\right)}{g\left(-q^{**}\right)} f\left(q^{**}\right)\frac{\alpha}{N}\\ &\quad\times\left[\begin{array}{c} q^{**}+(1-\mu)\beta_a+\mu\beta_c-\mu\frac{q^{**}+\beta_c}{1-G(-q^{**})}\\ +(1-\mu)\varepsilon-\mu\varepsilon+\mu\frac{\varepsilon}{1-G(-q^{**})}\end{array}\right].\end{aligned}$$

The derivative of $\lim_{\gamma\to\infty}\left[\mu MPV_c^{**}+(1-\mu) MPV_a^{**}\right]$ with respect to ε is proportional to

$$1-2\mu+\frac{\mu}{1-G\left(-q^{**}\right)} = 1-2\mu+\frac{\mu(1-\alpha)}{\tau-\alpha+\alpha\mu}.$$

Notice that

$$1-2\mu+\frac{\mu(1-\alpha)}{\tau-\alpha+\alpha\mu} > 0 \Leftrightarrow \tau-\alpha+(1-2(\tau-\alpha))\mu-2\alpha\mu^2 > 0.$$

This concave expression is positive both when $\mu=0$ and when $\mu=1$ (since $\alpha<1-\tau$), and thus it is positive for any $\mu\in[0,1]$. Therefore, $\lim_{\gamma\to\infty}\left[\mu MPV_c^{**}+(1-\mu) MPV_a^{**}\right]$ increases in ε, as required. ■

D Analysis of block trading

By definition, $\Pi^*(\beta) = \Pi(y^*(\beta),\beta)$, where $\Pi(y,\beta)$ is given by (24), $y^*(\beta)$ by (28), and $p^*(y,\beta) = \gamma y + v(\mathbb{E}[b], q^*(y,\beta))$. To ease the exposition we let $q^*(\beta) = q^*(y^*(\beta),\beta)$ and

$p^*(\beta) = p^*(y^*(\beta), \beta)$. Thus,

$$\begin{aligned}
\Pi^*(\beta) &= (\alpha + y^*(\beta))\, v(\beta, q^*(\beta)) - y^*(\beta)\, p^*(\beta) - \frac{\eta}{2}(y^*(\beta))^2 \\
&= \alpha v(\beta, q^*(\beta)) + y^*(\beta)(\beta - E[b])\, H(q^*(\beta)) - (\gamma + \eta/2)(y^*(\beta))^2 \\
&= \alpha v(\beta, q^*(\beta)) + \frac{1}{2}\frac{1}{2\gamma + \eta}\left[(\beta - \mathbb{E}[b])^2 H(q^*(\beta))^2 - MPV(y^*(\beta))^2\right]
\end{aligned}$$

The first term in the blockholder's equilibrium payoff is the value of his endowment given $q^*(\beta)$, whereas the second term represents the gains from trade.

Proposition 10. *There exist $\overline{\gamma} < \infty$ and $\overline{\eta} < \infty$ such that if $\gamma > \overline{\gamma}$ and $\eta > \overline{\eta}$, then the blockholder's equilibrium payoff strictly increases in β.*

Proof of Proposition 10. In the proof of Proposition 2, we show that if $\beta \neq G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})$ ($\beta \neq G^{-1}(\frac{1-\tau}{1-\alpha})$) and γ is large then $\beta \neq \beta_L(y^*(\beta))$ ($\beta \neq \beta_H(y^*(\beta))$). Then,

$$\begin{aligned}
\frac{\partial \Pi^*(\beta)}{\partial \beta} &= \frac{\partial \Pi(y^*(\beta), \beta)}{\partial y}\Big|_{y=y^*(\beta)} \cdot \frac{\partial y^*(\beta)}{\partial \beta} + \frac{\partial \Pi(y^*(\beta), \beta)}{\partial \beta} \\
&= 0 \cdot \frac{\partial y^*(\beta)}{\partial \beta} + \frac{\partial \Pi(y^*(\beta), \beta)}{\partial \beta} \\
&= (\alpha + y^*(\beta)) \frac{\partial v(\beta, q^*(y^*(\beta), \beta))}{\partial \beta} - y^*(\beta) \frac{\partial p^*(y^*(\beta), \beta)}{\partial \beta} \\
&= (\alpha + y^*(\beta)) \left[\frac{\partial v(\beta, q^*(y^*(\beta), \beta))}{\partial \beta} + \frac{\partial v(\beta, q^*(y^*(\beta), \beta))}{\partial q^*} \frac{\partial q^*(y^*(\beta), \beta)}{\partial \beta}\right] \\
&\quad - y^*(\beta) \frac{\partial v(\mathbb{E}[b], q^*(y^*(\beta), \beta))}{\partial q^*} \frac{\partial q^*(y^*(\beta), \beta)}{\partial \beta}
\end{aligned}$$

If $G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}) < \beta < G^{-1}(\frac{1-\tau}{1-\alpha})$ then for large γ we have $q^*(y, \beta) = -\beta$. Thus,

$$\begin{aligned}
\frac{\partial v(\beta, -\beta)}{\partial \beta} &= \frac{\partial}{\partial \beta} \int_{-\beta} (\theta + \beta) f(q)\, dq \\
&= H(-\beta) > 0
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial p^*(y^*(\beta), \beta)}{\partial \beta} &= \frac{\partial v(\mathbb{E}[b], -\beta)}{\partial \beta} \\
&= \frac{\partial}{\partial \beta} \int_{-\beta} (\theta + \mathbb{E}[b]) f(q)\, dq \\
&= -(\beta - \mathbb{E}[b]) f(-\beta)
\end{aligned}$$

Thus,

$$\frac{\partial \Pi^*(\beta)}{\partial \beta} = (\alpha + y^*(\beta))\, H(-\beta) + y^*(\beta)(\beta - \mathbb{E}[b])\, f(-\beta)$$

Recall that $\lim_{\gamma\to\infty} y^*(\beta) = 0$, and thus, for large γ we have $\frac{\partial \Pi^*(\beta)}{\partial \beta} \approx \alpha H(-\beta) > 0$, that is, the first order effect is on the blockholder endowment rather than on his gains from trade.

If $\beta < G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})$ $(G^{-1}(\frac{1-\tau}{1-\alpha}) < \beta)$ then $q^*(y^*(\beta),\beta) = \beta_L(y^*(\beta))$ $(q^*(y^*(\beta),\beta) = \beta_H(y^*(\beta)))$ does not depend on β directly, that is, $\frac{\partial q^*(y^*(\beta),\beta)}{\partial \beta} = 0$. And in this case,

$$\begin{aligned}\frac{\partial \Pi^*(\beta)}{\partial \beta} &= (\alpha + y^*(\beta)) \frac{\partial v(\beta, q^*(y^*(\beta),\beta))}{\partial \beta} \\ &= (\alpha + y^*(\beta)) H(q^*(y^*(\beta),\beta)) \\ &> 0\end{aligned}$$

Since $\Pi^*(\beta)$ is continuous in β when $\beta \in \left\{G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}), G^{-1}(\frac{1-\tau}{1-\alpha})\right\}$, $\Pi^*(\beta)$ increases globally in β. ■

Suppose a block of α shares is acquired by a bidder with a bias β who paid $\Pi^*(\beta) - \Delta\alpha$ where $\Delta > 0$. Parameter Δ captures the competitiveness of the block market, where smaller Δ implies more competitiveness. Recall the share price is given by $p^*(y^*(\beta)) = \gamma y^*(\beta) + v(\mathbb{E}[b], q^*(y^*(\beta),\beta))$, then the block premium is

$$P_B \equiv \Pi^*(\beta)/\alpha - \Delta - p^*(y^*(\beta),\beta) \tag{84}$$

Here, we replace $\Pi^*(\beta_R)$ in the text with $\Pi^*(\beta)$ and the discount blockholder R receives on the block price, $(1-\delta)(\Pi^*(\beta_R) - \Pi^*(\beta_I))$ with the constant Δ.

Proposition 11. *There exist $\overline{\gamma} < \infty$ and $\overline{\eta} < \infty$ such that if $\gamma > \overline{\gamma}$ and $\eta > \overline{\eta}$, then*

(i) *If $\beta \le \mathbb{E}[b]$ then the equilibrium block premium is strictly negative.*

(ii) *If $\beta > \mathbb{E}[b]$ then there exist $\alpha^* > 0$ and $\Delta^* > 0$ such that if $\alpha \in (0,\alpha^*)$ and $\Delta \in (0,\Delta^*)$ then the equilibrium block premium is strictly positive.*

Proof of Proposition 11. Notice that

$$\begin{aligned}\lim_{\gamma\to\infty} P_B &= \lim_{\gamma\to\infty} \Pi^*(\beta)/\alpha - \Delta - \lim_{\gamma\to\infty} p^*(y^*(\beta),\beta) \\ &= v(\beta, q^*(0,\beta)) - v(\mathbb{E}[b], q^*(0,\beta)) - \lim_{\gamma\to\infty}[\gamma y^*(\beta)] - \Delta \\ &= (\beta - \mathbb{E}[b]) H(q^*(0,\beta)) - \frac{1}{2}\left[\lim_{\gamma\to\infty} MPV(y^*) + (\beta - \mathbb{E}[b]) H(q^*(0,\beta))\right] - \Delta \\ &= \frac{1}{2}\left[(\beta - \mathbb{E}[b]) H(q^*(0,\beta)) - \lim_{\gamma\to\infty} MPV(y^*)\right] - \Delta\end{aligned}$$

Recall $\lim_{\gamma\to\infty} MPV(y^*) \ge 0$. Thus, if $\beta \le \mathbb{E}[b]$ then $\lim_{\gamma\to\infty} P_B < 0$. Suppose $\beta > E[b]$. Thus, $\frac{1}{2}(\beta - \mathbb{E}[b]) H(q^*(0,\beta)) - \Delta$ is bounded away from zero, and if $\lim_{\gamma\to\infty} MPV(y^*)$ is sufficiently small then $\lim_{\gamma\to\infty} P_B > 0$. There are three cases to consider:

1. If in addition $G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}) < \beta < G^{-1}(\frac{1-\tau}{1-\alpha})$ then $\lim_{\gamma\to\infty} MPV(y^*) = 0$.

2. If in addition $\beta < G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})$ then $q^*(0,\beta) = -G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})$, and

$$\lim_{\gamma\to\infty} MPV(y^*) = \frac{\tau}{1-\alpha}\frac{f(-G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}))}{g(G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}))}\frac{\alpha}{1-\alpha}(G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})-\beta)$$

Notice that $\lim_{\alpha\to 0}[\lim_{\gamma\to\infty} MPV(y^*)] = 0$, and hence, there exists α^* as required.

3. Suppose $\beta > G^{-1}(\frac{1-\tau}{1-\alpha})$. Then, $q^*(0,\beta) = -G^{-1}(\frac{1-\tau}{1-\alpha})$, and

$$\lim_{\gamma\to\infty} MPV(y^*) = \frac{1-\tau}{1-\alpha}\frac{f(-G^{-1}(\frac{1-\tau}{1-\alpha}))}{g(G^{-1}(\frac{1-\tau}{1-\alpha}))}\frac{\alpha}{1-\alpha}(\beta - G^{-1}(\frac{1-\tau}{1-\alpha})).$$

Notice that $\lim_{\alpha\to 0}[\lim_{\gamma\to\infty} MPV(y^*)] = 0$, and hence, there exists α^* as required.

■

D.1 Analysis of market for votes

In this section we extend our analysis by adding a separate market of voting rights to the baseline model. In our model, dispersed shareholders are never pivotal, and hence, would be willing to supply their votes for an arbitrarily small amount. Hereafter, we assume the price of a vote is zero, and that the blockholder will not buy any vote if he is indifferent. For simplicity, we assume that trades of votes and voting shares are simultaneous. That is, the blockholder submits an order to buy y voting shares and a fraction $\lambda \in [0,1]$ of all voting rights that are held by dispersed shareholders (through their ownership of voting shares post-trade). Then trades take place in both markets. As in the market for voting shares, we assume that the blockholder does not observe the bias of individual dispersed shareholders when trading votes, and thus, votes are sold by dispersed shareholders in proportion to their ownership of voting shares.[35]

For any given trade (y,λ), the blockholder owns a total of $\alpha + y + (1-\alpha-y)\lambda$ votes, and each share owned by dispersed shareholders has the right for $1-\lambda$ vote. Thus, the blockholder is pivotal for the vote outcome if and only if

$$\begin{aligned} s(q;y,q_e^*)(1-\lambda) &< \tau < s(q;y,q_e^*)(1-\lambda) + \alpha + y + (1-\alpha-y)\lambda \Leftrightarrow \\ \frac{\tau-\lambda}{1-\lambda} - \alpha - y &< s(q;y,q_e^*) < \frac{\tau}{1-\lambda} \end{aligned} \tag{85}$$

Notice that the RHS increases in λ and LHS decreases in λ. Since $s(q;y,q_e^*)$ is an increasing function of q, we have $\beta_l(y,q_e^*,\lambda) \equiv -s^{-1}\left(\frac{\tau}{1-\lambda};y,q_e^*\right)$ decreases in λ and $\beta_h(y,q_e^*,\lambda) \equiv -s^{-1}\left(\frac{\tau-\lambda}{1-\lambda} - \alpha - y; y, q_e^*\right)$ increases in λ. Let $\beta_L(y,\lambda)$ and $\beta_H(y,\lambda)$ be the solutions of $\beta_L =$

[35] Enabling the blockholder to discriminate and buy votes only from shareholders with a certain bias would further increase the blockholder's ability to influence the identity of the marginal voter. However, such discrimination may not be feasible when biases are unobserved.

$\beta_l(y, -\beta_L, \lambda)$ and $\beta_H = \beta_h(y, -\beta_H, \lambda)$, respectively, then the marginal voter is given by

$$-q^*(y,\lambda) = \begin{cases} \beta_L(y,\lambda) & \text{if } \beta < \beta_L(y,\lambda) \\ \beta & \text{if } \beta_L(y) < \beta < \beta_H(y,\lambda) \\ \beta_H(y,\lambda) & \text{if } \beta_H(y,\lambda) < \beta. \end{cases} \tag{86}$$

Notice that $\beta_L(y,\lambda)$ decreases in λ and $\beta_H(y,\lambda)$ increases in λ. Intuitively, the blockholder's access to the market for votes further increases his ability to influence the identity of the marginal voter. In particular, there is a wider region in which the blockholder is the marginal voter (and pivotal). Indeed, for any $y \in [-\alpha, 1-\alpha]$, $\beta_L(y,1) = -\bar{b}$ and $\beta_H(y,1) = \bar{b}$. Thus, without other constraints on vote-trading, the blockholder can use the market for votes to ensure he is the marginal voter.

Proof of Proposition ??. Suppose in equilibrium the blockholder obtains his ideal marginal voter, that is, $-q^*(y^*,\lambda^*) = -q_B^*(y^*)$, where $-q^*(y^*,\lambda^*)$ is defined by (86) and $-q_B^*(y^*)$ is defined by (42). Notice that if y^* satisfies $q^*(y^*,\lambda^*) = q_B^*(y^*)$ then $MPV(y^*) = 0$. Therefore, the FOC implies $MPC(y^*) = 0$, that is, $y^* = z(-q_B^*(y^*))$ where

$$z(-q^*) \equiv \frac{1}{2\gamma+\eta}(\beta - \mathbb{E}[b])\,H(q^*). \tag{87}$$

Let q_B^{**} be the solution of

$$\begin{aligned} -q^* &= \beta + \frac{z(-q^*)}{\alpha}(\beta - \mathbb{E}[b]) \Leftrightarrow \\ -q^* &= \beta + \frac{1}{\alpha}\frac{1}{2\gamma+\eta}H(q^*)(\beta - \mathbb{E}[b])^2. \end{aligned}$$

Then, it must be $-q^*(y^*,\lambda^*) = -q_B^{**}$. But notice that $-q_B^{**} > \beta$. According to (86), $-q^*(y^*,\lambda^*) > \beta$ implies $-q^*(y^*,\lambda^*) = \beta_L(y^*,\lambda^*)$. Therefore, $-q_B^{**} = \beta_L(z(-q_B^{**}),\lambda^*)$. Notice that for every y, $\beta_L(y,\lambda)$ spans $[\beta, \beta_L(y,0)]$. Therefore, the ideal marginal voter is obtained in equilibrium as long as there exists $\lambda \in [0,1]$ such $-q_B^{**} = \beta_L(z(-q_B^{**}),\lambda)$. Since $\beta_L(y,\lambda)$ is a decreasing function of λ, the ideal marginal voter is obtained in equilibrium if and only if $-q_B^{**} < \beta_L(z(-q_B^{**}),0)$. Notice that $\lim_{\gamma\to\infty} -q_B^{**} = \beta$ and $\lim_{\gamma\to\infty}\beta_L(z(-q_B^{**}),0) = G^{-1}\left(\frac{1-\alpha-\tau}{1-\alpha}\right)$. Therefore, $-q_B^{**} < \beta_L(z(-q_B^{**}),0)$ holds for large γ if and only if $\beta < G^{-1}\left(\frac{1-\alpha-\tau}{1-\alpha}\right)$, as required. ■