Online Appendix for "The Voting Premium"

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A Voting yields and capitalized voting premiums

The dual-class share premium is commonly computed as the relative price difference between voting and non-voting shares. Let $P_{t,v}$ be the price per voting share and $P_{t,nv}$ the price per non-voting shares in period t:[31]

$$\text{Dual class premium}_t = \frac{P_{t,v} - P_{t,nv}}{P_{t,v}}. \tag{73}$$

The dual-class premium captures the potentially infinite time horizon over which the owner of a block of voting rights enjoys control rights, which ends only if the firm ceases to exist, e.g., because of acquisitions or insolvencies, or when the two classes of shares are unified into one class. Hence, they represent the capitalized value of a the right to vote at all future shareholder meetings. The same is true for the block-trading premium, which is discussed below.

By contrast, the last three methods in our list measure the value of voting rights only for very limited periods of time ranging from three days to 57 days.[32] These time spans do not capture more than one shareholder meeting. Hence, these methods estimate a voting yield, which has the same dimension as a dividend yield. Let V_t represent the per-share dollar value of a voting right and D_t the dollar value of dividends per share, where the subscript indexes time. Then $D_t/P_{t-1,v}$ and $D_t/P_{t-1,nv}$ are the dividend yields of, respectively, the voting and the non-voting shares, and $V_t/P_{t-1,nv}$ is the voting yield. Let r_v be the constant per-period discount rate for the voting shares and r_{nv} the constant per-period discount rate for the non-voting shares, and assume the value of voting rights and dividends both grow at the same constant rate g, which allows us to calculate the value of voting shares and of non-voting shares using the Gordon growth formula:

$$P_{0,v} = \frac{D_1 + V_1}{r_v - g};\ P_{0,nv} = \frac{D_1}{r_{nv} - g}. \tag{74}$$

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[31] This statistic applies only if one class of shares has no voting rights and both classes have the same par value. It is appropriately adjusted when par values differ (Megginson (1990)) or when computing the value of control for firms that have two classes of voting shares, but different ratios of cash flow rights to voting rights; see, e.g., Zingales (1995). Bigelli and Croci (2013) Argue that many studies lack appropriate adjustments for differential dividends.

[32] Table 1 reports annualized figures if such figures are reported by the authors. Kalay, Karakas, and Pant (2014) and Gurun and Karakas (2020) construct non-voting shares synthetically from options with an average maturity of, respectively, 38 days and 57 days. the equity-lending method (Christoffersen et al. (2007); Aggarwal, Saffi, and Sturgess (2015)) investigates fees for lending shares around record dates, and the record-day trading method measures stock price drops in a 3-day trading window around record dates (Fos and Holderness (2020)).

Solving both expressions for g, equating them, and rearranging gives:

$$\frac{V_1}{P_{0,v}} = \frac{P_{0,v} - P_{0,nv}}{P_{0,nv}} \frac{D_1}{P_{0,v}} + r_v - r_{nv}. \tag{75}$$

Note, however, that numerous aspects are missing in this simple analysis, e.g., the time horizon for control may be finite because of possible mergers, stock unifications, or regulatory change. See, e.g., Goetzmann, Ukhov, and Spiegel (2002).If we assume that $r_v = r_{nv}$, we obtain:

$$\text{Voting yield} = \text{ Dual-class premium } \times \text{ Dividend yield.}$$

This relationship is broadly consistent with US data. A dual-class share premium of about 5%-10% (see Table 1), multiplied by a dividend yield of 3.7% (see Fama and French (2002) for the period 1951-2000) results in estimates of the voting yield of 0.18% to 0.37%, which compares well to estimates of the annualized voting yield from option replications (Kalay, Karakas, and Pant (2014): 0.16%; Kind and Poltera (2013): 0.37%). If we relax the assumption that $r_v = r_{nv}$, then from (75) we would expect a lower voting yield if the discount rate for voting shares is higher than that for non-voting shares, i.e., if investors consider votes to be riskier than dividends.

B Supplemental analysis for the baseline model

Lemma 3. *Let* $\underline{v} \equiv v_0 + \min\left\{-\bar{b}, \beta\right\}$ *and* $\overline{v} \equiv v_0 + \max\left\{\bar{b}, \beta\right\} + 1$. *Then,*

(i) *If* $\gamma > \frac{\overline{v}-\underline{v}}{1-\alpha}$ *then no dispersed shareholder short-sells the share in any equilibrium, that is,* $x^*(b) + 1 - \alpha > 0$ *for any* $b \in \left[-\bar{b}, \bar{b}\right]$.

(ii) *Suppose* $\alpha > 0$. *If* $\eta > \frac{4(\overline{v}-\underline{v})}{\alpha}$ *then the blockholder never short-sells the share in any equilibrium, that is,* $y^* + \alpha > 0$.

(iii) *If* $\alpha < \min\{\tau, 1-\tau\}$ *and* $\eta > \frac{2(\overline{v}-\underline{v})\min\{\tau,1-\tau\}}{(\min\{\tau,1-\tau\}-\alpha)^2}$ *then the blockholder never obtains a control stake or veto power in equilibrium, that is,* $\alpha + y^* < \min\{\tau, 1-\tau\}$.

Proof. The valuation of the share by any investor is bounded from above by $\overline{v}$ and from below by $\underline{v}$. Therefore, in any equilibrium, $|p^* - v(b, q^*)| < \overline{v} - \underline{v}$ and $|p^* - v(\beta, q^*)| < \overline{v} - \underline{v}$. Based on (7), $x(b, p^*) + 1 - \alpha > 0 \Leftrightarrow \gamma > \frac{p^* - v(b,q^*)}{1-\alpha}$. Therefore, requiring $\gamma > \frac{\overline{v}-\underline{v}}{1-\alpha}$ guarantees that in any equilibrium $x(b, p^*) + 1 - \alpha > 0$ as required by part (i).

Consider the blockholder. First, if the blockholder chooses $y^* < 0$ in equilibrium then it must be

$$\begin{aligned}
\Pi(y^*) &\geq \Pi(0) \Leftrightarrow \\
y^*\left[v(\beta, q^*(y^*)) - p^*(y^*)\right] - \frac{\eta}{2}y^{*2} &\geq \alpha\left[v(\beta, q^*(0)) - v(\beta, q^*(y^*))\right] \Rightarrow \\
-y^*(\overline{v} - \underline{v}) - \frac{\eta}{2}y^{*2} &\geq -\alpha(\overline{v} - \underline{v}) \Leftrightarrow \\
y^* &\geq -\frac{(\overline{v} - \underline{v}) + \sqrt{(\overline{v} - \underline{v})^2 + 2\alpha\eta(\overline{v} - \underline{v})}}{\eta}.
\end{aligned}$$

Therefore, assuming

$$-\frac{(\overline{v}-\underline{v})+\sqrt{(\overline{v}-\underline{v})^{2}+2\alpha\eta(\overline{v}-\underline{v})}}{\eta}>-\alpha\Leftrightarrow\eta>\frac{4(\overline{v}-\underline{v})}{\alpha}$$

guarantees that the blockholder will sell less than his entire endowment in any equilibrium, that is, $y^{*}>-\alpha$ as required by part (ii). Second, if the blockholder chooses $y^{*}>0$ in equilibrium then it must be

$$\begin{aligned}\Pi(y^{*}) &\geq \Pi(0)\Leftrightarrow\\ y^{*}\left[v(\beta,q^{*}(y^{*}))-p^{*}(y^{*})\right]-\frac{\eta}{2}y^{*2} &\geq \alpha\left[v(\beta,q^{*}(0))-v(\beta,q^{*}(y^{*}))\right]\Rightarrow\\ y^{*}(\overline{v}-\underline{v})-\frac{\eta}{2}y^{*2} &\geq -\alpha(\overline{v}-\underline{v})\Leftrightarrow\\ y^{*} &\leq \frac{\overline{v}-\underline{v}+\sqrt{(\overline{v}-\underline{v})^{2}+2\alpha\eta(\overline{v}-\underline{v})}}{\eta}.\end{aligned}$$

Therefore, assuming

$$\frac{\overline{v}-\underline{v}+\sqrt{(\overline{v}-\underline{v})^{2}+2\alpha\eta(\overline{v}-\underline{v})}}{\eta}<\min\{\tau,1-\tau\}-\alpha\Leftrightarrow\eta>\frac{2(\overline{v}-\underline{v})\min\{\tau,1-\tau\}}{(\min\{\tau,1-\tau\}-\alpha)^{2}}$$

guarantees that the blockholder will not obtain a stake larger than $\min\{\tau,1-\tau\}$ in any equilibrium, that is, $y^{*}+\alpha<\min\{\tau,1-\tau\}$ as required by part (iii). ■

B.1 Exit and a positive voting premium

Even if the blockholder has the power to increase his influence over the voting outcome and move the median voter closer to his bias β by buying additional shares, he may nevertheless choose to do the opposite: sell shares to dispersed shareholders and give up his influence over the voting outcome.

Corollary 1. *Suppose $\beta<\mathbb{E}[b]$ and $\alpha\in(0,\alpha^{*})$ for some $\alpha^{*}>0$. There exist $\bar{\gamma}$ and $\bar{\eta}$ such that if $\gamma>\bar{\gamma}$ and $\eta>\bar{\eta}$, then $y^{*}<0$ and $MPV(y^{*})>0$.*

Intuitively, if $\beta<\mathbb{E}[b]$, cash flow considerations lead the blockholder to sell ($MPC<0$). However, since selling moves the median voter away from the blockholder's preferred location, it leads to a loss on his endowment α. If the blockholder's endowment is small, as under the conditions of Corollary 1, the negative effect on his endowment is not sufficient to outweigh the benefits from selling, resulting in $y^{*}<0$. However, since selling diminishes the blockholder's ability to influence the vote outcome, he demands a premium from the dispersed shareholders, leading to a positive MPV. Thus, the voting premium can be positive even when the blockholder sells shares, i.e., when the ownership structure becomes less concentrated.

Proof of Corollary 1. Based on Proposition 2 and its proof, if $G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})<\beta<G^{-1}(\frac{1-\tau}{1-\alpha})$ then $MPV(y^{*})=0$. If $\beta<G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})$ then based on case 1.b in the proof of Proposition 2,

$MPV(y^*) > 0$ and the FOC holds and in the limit,

$$(\beta - \mathbb{E}[b]) H(q^*(0)) + \frac{\tau}{1-\alpha}\frac{\alpha}{1-\alpha}\frac{f(-G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}))}{g(G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}))}(G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}) - \beta) = \lim_{\gamma\to\infty} 2\gamma y^*.$$

Notice that if $\beta < \mathbb{E}[b]$ and α is sufficiently small, then the LHS is negative, and hence $\lim_{\gamma\to\infty} 2\gamma y^* < 0$. This implies that for large enough γ it must be $y^* < 0$. If $\beta > G^{-1}(\frac{1-\tau}{1-\alpha})$ then based on case 2.b in the proof of Proposition 2, $MPV(y^*) > 0$ and the FOC holds and in the limit,

$$(\beta - \mathbb{E}[b]) H(q^*(0)) + \frac{1-\tau}{1-\alpha}\frac{\alpha}{1-\alpha}\frac{f(-G^{-1}(\frac{1-\tau}{1-\alpha}))}{g(G^{-1}(\frac{1-\tau}{1-\alpha}))}(\beta - G^{-1}(\frac{1-\tau}{1-\alpha})) = \lim_{\gamma\to\infty} 2\gamma y^*.$$

Notice that if $\beta < \mathbb{E}[b]$ and α is sufficiently small, then the LHS is negative, and hence $\lim_{\gamma\to\infty} 2\gamma y^* < 0$. This implies that for large enough γ it must be $y^* < 0$, which completes the proof. ■

B.2 Conflicts of interest and homogeneous shareholders

It is often argued that the voting premium arises if there is a conflict between the blockholder and all dispersed shareholders, so that the blockholder gains what the dispersed shareholders lose (which could be interpreted as the blockholder's private benefits of control). In the context of our model, such a conflict does not by itself generate a voting premium. To see this, suppose that the distribution G of dispersed shareholders' private values is concentrated around its mean, and this mean is different from the bias of the blockholder, $\mathbb{E}[b] \neq \beta$. In such a setting, there are no dispersed shareholders who are close to the blockholder, so any move of the median voter towards the blockholder moves the median voter away from all dispersed shareholders, i.e., there is a conflict between the blockholder and *all* dispersed shareholders. The next result shows that such a conflict does not automatically give rise to a voting premium.

Proposition 6 (Conflicts of interest). *Suppose $\mathbb{E}[b] \neq \beta$. Consider a mean-preserving parametrization δ such that as $\delta \to 0$, the cdf $G(b;\delta)$ becomes more concentrated around the mean $\mathbb{E}[b]$. Then, $\lim_{\delta\to 0} MPV(y^*(\delta)) = 0$.*

In Proposition 6, the dispersion of private values among small shareholders becomes second order relative to the difference between them and the blockholder. Then, as the heterogeneity among small shareholders vanishes, the blockholder loses his ability to influence the identity of the median voter through his trades. Accordingly, his control motive for buying the shares vanishes, and so does the voting premium.

Proof of Proposition 6. Recall

$$s_{y,-z}(-z) = (1 - G(z))(1 - \alpha - y) + \frac{1}{\gamma} G(z)(\mathbb{E}[b] - \mathbb{E}[b|b < z]) H(-z).$$

The set of equations (41) and (42) can be written as

$$\begin{aligned} s_{y,-\beta_L}(-\beta_L) &= \tau \\ s_{y,-\beta_H}(-\beta_H) &= \tau - \alpha - y. \end{aligned}$$

Let $z(\delta)$ be a sequence. Then,

$$\lim_{\delta \to 0} s_{y,-z(\delta)}(-z(\delta)) = (1 - \alpha - y) \times \left(1 - \lim_{\delta \to 0} G(z(\delta);\delta)\right).$$

If $\lim_{\delta\to 0} z(\delta) > \mathbb{E}[b]$, then $\lim_{\delta\to 0} G(z(\delta);\delta) = 1$ and $\lim_{\delta\to 0} s_{y,-z(\delta)}(-z(\delta)) = 0$. And if $\lim_{\delta\to 0} z(\delta) < \mathbb{E}[b]$, then $\lim_{\delta\to 0} G(z(\delta);\delta) = 0$ and $\lim_{\delta\to 0} s_{y,-z(\delta)}(-z(\delta)) = 1 - \alpha - y$. As long as $\tau \in (0,1)$, the solutions of (41) and (42) do not exist in the limit of $\delta \to 0$. Therefore, it must be $\lim_{\delta\to 0} z(\delta) = \mathbb{E}[b]$. Since $\beta_L(y;\delta)$ solves $s_{y,-\beta_L}(-\beta_L) = \tau$ and $\lim_{\delta\to 0} s_{y,-z(\delta)}(-z(\delta)) = (1-\alpha-y) \times (1 - \lim_{\delta\to 0} G(z(\delta);\delta))$, then $\lim_{\delta\to 0} \beta_L(y;\delta) = \mathbb{E}[b]$ and it converges at a rate that satisfies $1 - \frac{\tau}{1-\alpha-y} = \lim_{\delta\to 0} G(\beta_L(y;\delta);\delta)$. Similarly, since $\beta_H(y;\delta)$ solves $s_{y,-\beta_H}(-\beta_H) = \tau-\alpha-y$ and $\lim_{\delta\to 0} s_{y,-z(\delta)}(-z(\delta)) = (1-\alpha-y)\times(1 - \lim_{\delta\to 0} G(z(\delta);\delta))$, then $\lim_{\delta\to 0} \beta_H(y) = \mathbb{E}[b]$ and it converges at a rate that satisfies $1-\frac{\tau-\alpha-y}{1-\alpha-y} = \lim_{\delta\to 0} G(\beta_H(y;\delta);\delta)$.

Finally, since $\lim_{\delta\to 0} \beta_H(y) = \lim_{\delta\to 0} \beta_L(y) = \mathbb{E}[b]$ for any $y \in (-\alpha, \min\{\tau, 1-\tau\} - \alpha)$, then $\lim_{\delta\to 0} \frac{\partial(-q^*(y))}{\partial y} = 0$, which implies $MPV(y) = 0$. ■

C Extensions

In this section, we discuss several extensions of the baseline model. In Section C.1, we analyze a separate market for votes. In Section C.2, we examine the influence premium and contrast it to the voting premium. In Section C.3 we consider the case of multiple blockholders, and in Section C.4 we extend the model by an initial stage in which blockholders trade with each other.

C.1 Analysis of the market for votes

In this section, we extend our analysis by adding a separate market of voting rights. Since dispersed shareholders are never pivotal for the voting outcome, they are willing to supply their votes for an arbitrarily small price. Therefore, we assume that the price of a vote is zero. We also assume that vote trading involves no transaction costs and that the blockholder will not buy any votes if he is indifferent. For simplicity, we assume that trades of votes and voting shares are simultaneous. That is, the blockholder submits an order to buy y voting shares and a fraction $\lambda \in [0,1]$ of all voting rights that are held by dispersed shareholders (through their ownership of voting shares post-trade). Then trades take place in both markets. As in the market for voting shares, we assume that the blockholder does not observe the bias of individual dispersed shareholders when trading votes, and thus, votes are sold by dispersed shareholders in proportion to their ownership of voting shares.[33]

[33] Enabling the blockholder to discriminate and buy votes only from shareholders with a certain bias would further increase the blockholder's ability to influence the identity of the median voter. However, such discrimination may not be feasible when biases are unobserved.

For any given trade (y, λ), the blockholder owns a total of $\alpha + y + (1 - \alpha - y)\lambda$ votes, and each share owned by dispersed shareholders has the right for $1 - \lambda$ vote. Thus, the blockholder is pivotal for the vote outcome if and only if

$$\begin{aligned} s_{y,q_e^*}(q)(1-\lambda) &< \tau < s_{y,q_e^*}(q)(1-\lambda) + \alpha + y + (1-\alpha-y)\lambda \Leftrightarrow \\ \frac{\tau-\lambda}{1-\lambda} - \alpha - y &< s_{y,q_e^*}(q) < \frac{\tau}{1-\lambda} \end{aligned} \tag{76}$$

Notice that the RHS increases in λ and LHS decreases in λ. Since $s_{y,q_e^*}(q)$ is an increasing function of q, we have $\beta_l(y, q_e^*, \lambda) \equiv -s_{y,q_e^*}^{-1}\left(\frac{\tau}{1-\lambda}\right)$ decreases in λ and $\beta_h(y, q_e^*, \lambda) \equiv -s_{y,q_e^*}^{-1}\left(\frac{\tau-\lambda}{1-\lambda} - \alpha - y\right)$ increases in λ. Let $\beta_L(y, \lambda)$ and $\beta_H(y, \lambda)$ be the solutions of $\beta_L = \beta_l(y, -\beta_L, \lambda)$ and $\beta_H = \beta_h(y, -\beta_H, \lambda)$, respectively. Then the median voter is given by

$$-q^*(y,\lambda) = \begin{cases} \beta_L(y,\lambda) & \text{if } \beta < \beta_L(y,\lambda) \\ \beta & \text{if } \beta_L(y) < \beta < \beta_H(y,\lambda) \\ \beta_H(y,\lambda) & \text{if } \beta_H(y,\lambda) < \beta. \end{cases} \tag{77}$$

Notice that $\beta_L(y, \lambda)$ decreases in λ and $\beta_H(y, \lambda)$ increases in λ. Intuitively, the blockholder's access to the market for votes further increases his ability to influence the identity of the median voter. In particular, there is a wider region in which the blockholder is the median voter (and pivotal). Indeed, for any $y \in [-\alpha, 1-\alpha]$, $\beta_L(y, 1) = -\bar{b}$ and $\beta_H(y, 1) = \bar{b}$. Thus, without other constraints on vote-trading, the blockholder can use the market for votes to ensure he is the median voter.

We next examine two additional questions. We first ask how the blockholder would choose the median voter if he could make this choice without any constraints. Next, we ask whether the blockholder can achieve his desired median voter through the market for votes.

C.1.1 The ideal median voter

We first analyze how the blockholder would choose the median voter. The blockholder's ideal median voter, denoted by $-q_B^*$, is obtained when the blockholder has no incentives to further influence the median voter. Based on (26), this happens whenever the sum of the endowment and net trading benefits is zero, which gives

$$-q_B^*(y) = \beta + \frac{y}{\alpha}(\beta - \mathbb{E}[b]). \tag{78}$$

The expression has two components. If the blockholder does not trade ($y = 0$), his ideal median voter is $-q_B^* = \beta$, which maximizes the long-term value of his endowment. However, the more important are the blockholder's short-term trading considerations relative to his endowment, $\frac{y}{\alpha}$, the further away from β is his ideal median voter. The wedge between the blockholder's bias and the average dispersed shareholder's bias, which captures the potential gains from trade, determines the deviation of the blockholder's ideal median voter from β.

C.1.2 The ideal median voter and vote trading

In our baseline model, voting and cash flow rights are always bundled in one security. Hence, even if the blockholder could choose his ideal median voter by picking the appropriate y, he would generally choose not to do so because of cash flow considerations, so the optimal trade y^* generally does not lead to the ideal median voter, $-q_B^*(y^*) \neq -q^*(y^*)$. Therefore, it is interesting to explore whether the blockholder can obtain his ideal median voter if votes can be separated from cash flow rights. Recall that we make the following assumptions about the market for votes: we assume that the price of a vote is zero, that vote trading involves no transaction costs, and that the votes are sold by dispersed shareholders in proportion to their ownership of the voting shares. These assumptions likely overstate the blockholder's ease of access to the market for votes.[34] Nevertheless, the next result shows that the blockholder cannot always obtain his ideal median voter in equilibrium.

Proposition 7. *Suppose the blockholder has access to the market of votes. There exist $\overline{\gamma} < \infty$ and $\overline{\eta} < \infty$ such that if $\gamma > \overline{\gamma}$ and $\eta > \overline{\eta}$, then the blockholder obtains his ideal median voter if and only if $\beta < G^{-1}\left(\frac{1-\alpha-\tau}{1-\alpha}\right)$.*

To understand the intuition, first note that the blockholder's ideal median voter is typically more activist than the blockholder, $-q_B^*(y^*) > \beta$. Indeed, if $\beta > \mathbb{E}[b]$ ($\beta < \mathbb{E}[b]$), then dispersed shareholders on average dislike (like) the proposal more than the blockholder, and the blockholder would benefit from buying (selling) shares. By pushing the median voter to have a bias greater than β, the blockholder decreases (increases) the valuation of dispersed shareholders, and therefore, the price at which he can buy (sell) shares. Thus, the net trading benefits push $-q_B^*(y)$ to be greater than β.

However, as Proposition 7 demonstrates, the market for votes does not allow the blockholder to achieve this ideal median voter if the blockholder is already sufficiently activist, $\beta > G^{-1}\left(\frac{1-\alpha-\tau}{1-\alpha}\right)$. Intuitively, while vote-buying increases the blockholder's influence on the identity of the median voter, it does so in a very specific way: it always pushes the median voter closer to the blockholder. This is because when the blockholder casts his vote, his gains from trade are sunk, so he always votes to maximize the value of his position. Without a commitment to do otherwise, the accumulation of disproportional voting rights can only push the bias of the median voter even closer to β. When the blockholder is less activist than the dispersed shareholders, he can obtain a more activist ideal median voter by rationing the amount of votes he buys from the dispersed shareholders, who are more activist. However, when the blockholder is relatively activist, the best he can do is to buy enough votes to ensure that he is the median voter, but he cannot use the market for votes to select a median voter who is more activist than himself.

Proposition 7 has the following interesting implication. A blockholder whose bias toward the proposal is relatively small can use shareholder voting to fulfill his agenda. However, if the blockholder is highly motivated and his bias toward the proposal is relatively large, the market for votes would be insufficient, and the blockholder could potentially benefit from other channels of influence, such as lobbying proxy advisory firms and regulators; engaging in behind-the-scenes discussions with management; or launching media campaigns to put pressure on the firm. Thus, even when shareholder voting is amplified by the market for votes, other mechanisms of corporate governance could continue to play a role, an observation consistent with the evidence that institutional investors use a variety of channels to exert corporate influence (e.g., McCahery, Sautner, and Starks (2016)).

[34] See Christoffersen et al. (2007) for a discussion of the market for equity lending. Their results suggest that the ease of access assumed here is realistic.

Proof of Proposition 7. Suppose in equilibrium the blockholder obtains his ideal median voter, that is, $-q^*(y^*,\lambda^*) = -q_B^*(y^*)$, where $-q^*(y^*,\lambda^*)$ is defined by (77) and $-q_B^*(y^*)$ is defined by (78). Notice that if y^* satisfies $q^*(y^*,\lambda^*) = q_B^*(y^*)$ then $MPV(y^*) = 0$. Therefore, the FOC implies $MPC(y^*) = 0$, that is, $y^* = z(-q_B^*(y^*))$ where

$$z(-q^*) \equiv \frac{1}{2\gamma+\eta}(\beta - \mathbb{E}[b])\,H(q^*). \tag{79}$$

Let q_B^{**} be the solution of

$$\begin{aligned}
-q^* &= \beta + \frac{z(-q^*)}{\alpha}(\beta - \mathbb{E}[b]) \Leftrightarrow \\
-q^* &= \beta + \frac{1}{\alpha}\frac{1}{2\gamma+\eta}H(q^*)(\beta - \mathbb{E}[b])^2 .
\end{aligned}$$

Then, it must be $-q^*(y^*,\lambda^*) = -q_B^{**}$. But notice that $-q_B^{**} > \beta$. According to (77), $-q^*(y^*,\lambda^*) > \beta$ implies $-q^*(y^*,\lambda^*) = \beta_L(y^*,\lambda^*)$. Therefore, $-q_B^{**} = \beta_L(z(-q_B^{**}),\lambda^*)$. Notice that for every y, $\beta_L(y,\lambda)$ spans $[\beta, \beta_L(y,0)]$. Therefore, the ideal median voter is obtained in equilibrium as long as there exists $\lambda \in [0,1]$ such $-q_B^{**} = \beta_L(z(-q_B^{**}),\lambda)$. Since $\beta_L(y,\lambda)$ is a decreasing function of λ, the ideal median voter is obtained in equilibrium if and only if $-q_B^{**} < \beta_L(z(-q_B^{**}),0)$. Notice that $\lim_{\gamma\to\infty} -q_B^{**} = \beta$ and $\lim_{\gamma\to\infty} \beta_L(z(-q_B^{**}),0) = G^{-1}\left(\frac{1-\alpha-\tau}{1-\alpha}\right)$. Therefore, $-q_B^{**} < \beta_L(z(-q_B^{**}),0)$ holds for large γ if and only if $\beta < G^{-1}\left(\frac{1-\alpha-\tau}{1-\alpha}\right)$, as required. ■

C.2 The influence premium

In practice, if the company's management makes decisions taking into account the preferences of its shareholder base, then shareholders may influence corporate decision making even though they do not have formal control rights. The ability to influence decisions in such cases may give rise to an "influence premium" on the share price.

To study the influence premium, we consider a variant of the baseline model in which the proposal is decided not by a shareholder vote, but rather by the company's management, which observes signal q before making its decision on the proposal. All other aspects of the model remain unchanged. We assume that the management trades-off its own personal agenda with the average value of the post-trade shareholder base, which we denote by $W_{post\_trade}(d,\theta)$. Specifically, the management's payoff is

$$\omega W_{post\_trade}(d,\theta) + (1-\omega)\,v(d,\theta,b_m), \tag{80}$$

where b_m is the management's bias toward the proposal, and $\omega \in (0,1)$ captures the extent to which the management prioritizes the interests of current shareholders over its own interests.

Since the management cares about the average value of the post-trade shareholder base, changes in the composition of the shareholder base affect its decision on the proposal. From the perspective of the management, the preferences of the post-trade shareholder base can be summarized by the identity of the average post-trade shareholder. In Proposition 8 below, we show that if the blockholder trades y shares and the management is expected to approve the proposal if and only if $q > q_m^*$ for some cutoff q_m^*, then the average post-trade shareholder has a bias of

$$b_{post\_trade}\left(y, q_m^*\right) = \left(\alpha\beta + (1-\alpha)\,\mathbb{E}\left[b\right]\right) + y\left(\beta - \mathbb{E}\left[b\right]\right) + \frac{1}{\gamma} H\left(q_m^*\right) \mathbb{V}\left(b\right), \tag{81}$$

where $\mathbb{V}(b)$ captures the cross-sectional variance of the initial dispersed shareholders' biases. Intuitively, the first term, $\alpha\beta + (1-\alpha)\,\mathbb{E}[b]$, is the bias of the average pre-trade shareholder base; the second term captures the effect of trades between dispersed shareholders and the blockholder; and the third term captures the effect of trades among dispersed shareholders.

It follows that as long as $\beta \neq \mathbb{E}[b]$, the blockholder's trades affect the preferences of the average post-trade shareholder base, and thereby indirectly influence the management's decisions. The anticipation of being able to influence the management gives rise to an *influence premium* on the firm's shares. Similar to the voting premium, the equilibrium influence premium is proportional to the MPV as given by (19). However, different from the voting premium, the voting decision rule $q^*(y)$ is replaced by the management's cutoff $q_m^*(y)$, which depends on the bias of the average post-trade shareholder, $b_{post\_trade}(y, q_m^*)$. Intuitively, while in the baseline model the blockholder's trades affect the decision on the proposal by changing the identity of the *median voter*, $-q^*(y)$, here the effect is channeled through changes in the identity of the *average shareholder*.

Since the median voter and the average shareholder are different from each other, there is a wedge between the voting premium and the influence premium. In particular, if decisions are made by voting, then once the blockholder buys sufficiently many shares, he becomes the median voter and cannot further influence the median voter's identity (Proposition 1), so the voting premium is zero. By contrast, if decisions are made by the management, then as long as $\beta \neq \mathbb{E}[b]$, and even if the blockholder's stake is quite large, his incremental buying always pushes the average shareholder bias closer to β, and as such, further influences the management's decision. As a result, the influence premium can be strictly larger than the corresponding voting premium, even if the company's management puts an arbitrarily small weight ω on maximizing shareholder value.

The next proposition formally demonstrates the arguments presented above.

Proposition 8. *Suppose the blockholder has an endowment* $\alpha > 0$.

(*i*) *In the setting of Section C.2, there exist* $\overline{\gamma} < \infty$ *and* $\overline{\eta} < \infty$ *such that if* $\gamma > \overline{\gamma}$ *and* $\eta > \overline{\eta}$, *the equilibrium exists and is unique. This equilibrium is characterized as in Proposition 2, except that the decision rule* $q^*(y^*)$ *is replaced by the management's decision rule* $q_m^*(y^*)$: *the proposal is accepted if and only if* $q > q_m^*(y^*)$, *where* $q_m^*(y)$ *is given by the unique solution of*

$$-q_m^* = \omega b^*_{post\_trade}\left(y, q_m^*\right) + (1-\omega)\, b_m, \tag{82}$$

where

$$b^*_{post\_trade}\left(y, q_m^*\right) = \alpha\beta + (1-\alpha)\,\mathbb{E}\left[b\right] + y\left(\beta - \mathbb{E}\left[b\right]\right) + \frac{1}{\gamma} H\left(q_m^*\right) \mathbb{V}\left(b\right).$$

(*ii*) *If* $\beta \neq \mathbb{E}[b]$ *and* $\beta \in \left(G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}), G^{-1}(\frac{1-\tau}{1-\alpha})\right)$, *then the voting premium is zero but the influence premium is strictly positive.*

Proof of Proposition 8. (i) Suppose shareholders, including the blockholder, trade based on the expectations that the management will follow a decision rule $q_e^*(y)$. Then, the trading strategies are the same as described in the baseline model. Therefore, the average value of the post-trade shareholder base conditional on y and q is

$$\begin{aligned} W_{post\_trade}(d, q; y, q_e^*(y)) &= (\alpha + y)\, v(d, q, \beta) + \int_{-\bar{b}}^{\bar{b}} (1 - \alpha + x(b, y, p_e^*))\, v(d, q, b)\, g(b)\, db \\ &= v\left(d, q, b^*_{post\_trade}(y, q_e^*(y))\right) \end{aligned}$$

where

$$\begin{aligned} b^*_{post\_trade}(y, q_e^*(y)) &= (\alpha + y)\beta + \int_{-\bar{b}}^{\bar{b}} (1 - \alpha + x(b, y, p_e^*))\, b g(b)\, db \\ &= (\alpha + y)\beta + \int_{-\bar{b}}^{\bar{b}} \left[\left(1 - \alpha + \frac{v(b, q_e^*(y)) - p_e^*}{\gamma}\right) b\right] g(b)\, db \\ &= (\alpha + y)\beta + \int_{-\bar{b}}^{\bar{b}} \left[\left(1 - \alpha + \frac{v(b, q_e^*(y)) - v(\mathbb{E}[b], q_e^*(y)) - \gamma y}{\gamma}\right) b\right] g(b)\, db \\ &= \alpha\beta + (1 - \alpha)\mathbb{E}[b] + y(\beta - \mathbb{E}[b]) + \frac{1}{\gamma}\int_{-\bar{b}}^{\bar{b}} [(v(b, q_e^*(y)) - v(\mathbb{E}[b], q_e^*(y)))\, b]\, g(b)\, db \\ &= \alpha\beta + (1 - \alpha)\mathbb{E}[b] + y(\beta - \mathbb{E}[b]) + \frac{1}{\gamma} H(q_e^*(y))\, \mathbb{V}(b), \end{aligned}$$

where

$$\mathbb{V}(b) \equiv \int_{-\bar{b}}^{\bar{b}} (b - \mathbb{E}[b])\, b g(b)\, db$$

captures the cross-sectional variance of the initial dispersed shareholders' biases. Thus, given linearity of $v(d, \theta, b)$ in b, the management's decision on the proposal for a given realization of q is aimed to maximize

$$v\left(d, q, \omega b^*_{post\_trade}(y, q_e^*(y)) + (1 - \omega) b_m\right),$$

which implies that the management approves the proposal if and only if $q > q_m^*(y)$, where

$$-q_m^*(y) = \omega b^*_{post\_trade}(y, q_e^*(y)) + (1 - \omega) b_m.$$

In equilibrium, the expectations of shareholders must be consistent with the management's decision making rule, and hence $q_m^*(y)$ must solve

$$-q_m^* = \omega b^*_{post\_trade}(y, q_m^*) + (1 - \omega) b_m. \tag{83}$$

Notice that given y,

$$\lim_{\gamma \to \infty} b^*_{post\_trade}(y, q_e^*(y)) = \alpha\beta + (1 - \alpha)\mathbb{E}[b] + y(\beta - \mathbb{E}[b]),$$

and thus, if γ is large, then a solution always exists and is unique. Given $q_b^*(y)$, the blockholder solves the same problem as in the baseline model, with the exception that $q^*(y)$ is replaced by $q_m^*(y)$. Notice that

$$\frac{\partial\left(-q_m^*(y)\right)}{\partial y}=\frac{\omega\left(\beta-\mathbb{E}[b]\right)}{1-\omega\frac{\mathbb{V}(b)}{\gamma}f\left(q_m^*(y)\right)}, \tag{84}$$

and as long as $\beta\neq\mathbb{E}[b]$, we have $\frac{\partial(-q_m^*(y))}{\partial y}\neq 0$.

(ii) As in the baseline model, the influence premium is determined by the blockholder's marginal benefit from influencing the management's decision rule $q_m^*(y)$ through his trades:

$$MPV_{\text{influence}}(y)=\frac{\partial\left(-q_m^*(y)\right)}{\partial y}\left[\alpha\left(q_m^*(y)+\beta\right)+y\left(\beta-\mathbb{E}[b]\right)\right]f\left(q_m^*(y)\right),$$

and the optimal trade y^* satisfies $\lim_{\gamma\to\infty}y^*=0$. Thus,

$$\begin{aligned}
\lim_{\gamma\to\infty}-q_m^*(y^*) &= \alpha\beta+(1-\alpha)\mathbb{E}[b]\\
\lim_{\gamma\to\infty}\frac{\partial\left(-q_m^*(y)\right)}{\partial y}\Big|_{y=y^*} &= \omega\left(\beta-\mathbb{E}[b]\right)\\
\lim_{\gamma\to\infty}MPV_{\text{influence}}(y^*) &= \omega\alpha(1-\alpha)\left(\beta-\mathbb{E}[b]\right)^2 f\left(-\alpha\beta-(1-\alpha)\mathbb{E}[b]\right)>0.
\end{aligned}$$

Thus, as long as $\beta\neq\mathbb{E}[b]$, we have $MPV_{\text{influence}}(y^*)>0$ for large γ, and hence the influence premium is strictly positive.

Finally, recall from Proposition 2 that if $\beta\in\left(G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}),G^{-1}(\frac{1-\tau}{1-\alpha})\right)$, then $MPV(y^*)=0$, so the voting premium is zero. ■

C.3 Multiple blockholders

In this section, we consider an extension of the baseline model to the case of multiple blockholders. We analyze two scenarios, one in which blockholders have identical goals and one in which they have conflicting goals.

Specifically, suppose there are $N\geq 2$ blockholders, where each blockholder is endowed with $\alpha_i=\frac{\alpha}{N}$ shares and faces a trading cost η. In the first scenario, we assume that all blockholders have the same bias β. Hence, blockholders compete when trading shares, but they apply the same decision rule when voting on the proposal. We solve for the symmetric equilibrium in which the optimal trades for all blockholders are the same, y^*. Furthermore, we maintain the same assumptions as in Proposition 2 to ensure that the equilibrium exists and is unique.[35] We provide the full analysis of this extension in Section C.3.1 below and summarize the main conclusions here.

The share price with N blockholders is given by

$$p^*(N)=v\left(b_{MT}(N),q^*(Ny^*)\right)+\frac{\gamma}{\gamma(N+1)+\eta}MPV(Ny^*), \tag{85}$$

where $q^*(\cdot)$ is given by (15), $MPV(\cdot)$ is given by (26), and $b_{MT}(N)$ is defined in Section C.3.1 below.

[35] We also impose conditions on γ and η to guarantee that the trade y_i of each blockholder satisfies $y_i>-\frac{\alpha}{N}$.

As the number of blockholders increases, the share price puts an increasingly smaller weight on voting considerations and a larger weight on cash flow considerations. Moreover, the marginal trader $b_{MT}(N)$ converges to β, the bias of the blockholders. Intuitively, blockholders compete with each other on buying shares. As in Cournot competition, they do not fully internalize the effect of their own trades on their peers, and as N increases, this effect becomes more and more significant (see Kyle (1989) and Edmans and Manso (2011) for related effects). Each blockholder fully bears the costs of his trades, but the benefits from affecting the median voter accrue to all other blockholders as well. In the limit, competition among blockholders reduces their marginal propensity to buy votes and hence the voting premium to zero.

The conclusion that the voting premium decreases with the number of blockholders depends on the assumption that blockholders are homogeneous and, accordingly, free ride on each other's efforts to move the median voter. Hence, in another extension, we consider the scenario in which the blockholders have conflicting interests. Suppose μN blockholders have bias $\beta_c \leq -\Delta$, and $(1-\mu)N$ blockholders have bias $\beta_a \geq \Delta$, where $\mu \in (0,1)$ is such that both μN and $(1-\mu)N$ are integers. Blockholders with bias β_c always (i.e., regardless of the realization of signal q) vote against the proposal, and blockholders with bias β_a always vote for the proposal.

The complete analysis of the symmetric equilibrium is presented in Section C.3.2 below. We show that blockholders with bias β_a and those with bias β_c have different MPVs, which depend on their biases and also result in different optimal trades. As the number N of blockholders increases, the bias of the marginal trader $b_{MT}(N)$ now converges to $\overline{\beta} \equiv (1-\mu)\beta_a + \mu\beta_c$, i.e., the weighted average of the biases of the two groups of blockholders.

The key observation from this extension is that if blockholders have stronger conflicts of interests, they value their voting rights more. Specifically, as blockholders become more extreme such that β_a increases by ε and β_c decreases by ε, then both the aggregate MPV, $\mu MPV_c^{**} + (1-\mu) MPV_a^{**}$, and the voting premium increase. This is true even if the average bias of the blockholders remains unchanged (e.g., $\mu = \frac{1}{2}$ and the biases become more extreme by the same amount). Intuitively, the marginal propensity to buy votes is positive both for blockholders biased in favor and those biased against the proposal, since both are trying to move the median voter in their preferred direction, which opposes the preferred direction of the other type. As the biases become more extreme, the incentives to do so increase, which is reflected in a higher voting premium.

Overall, the conclusion of these two extensions is that the magnitude of the voting premium crucially depends on whether blockholders have the same or conflicting objectives.

C.3.1 Homogenous blockholders

Assume there are $N \geq 2$ blockholders, each blockholder is endowed with $\alpha_i = \frac{\alpha}{N}$ shares. All blockholders face the same trading cost η and have the same bias β.

Proposition 9. *There exist $\overline{\gamma} < \infty$ and $\overline{\eta} < \infty$ such that if $\gamma > \overline{\gamma}$ and $\eta > \overline{\eta}$, then the equilibrium exists and is unique. The equilibrium trade of each blockholder, denoted by y^*, satisfies*

$$Ny^* = \frac{N}{\gamma(N+1)+\eta}(\beta - \mathbb{E}[b])H(q^*(Ny^*)) + \frac{1}{\gamma(N+1)+\eta}MPV(Ny^*), \qquad (86)$$

and the share price is given by

$$p^*(N) = v(b_{MT}(N), q^*(Ny^*)) + \frac{\gamma}{\gamma(N+1)+\eta} MPV(Ny^*),$$

where $q^*(\cdot)$ is given by (15), $MPV(\cdot)$ is given by (26), and

$$b_{MT}(N) = \frac{\gamma+\eta}{\gamma(N+1)+\eta}\mathbb{E}[b] + \frac{\gamma N}{\gamma(N+1)+\eta}\beta. \tag{87}$$

Proof of Proposition 9. We denote the trade of blockholder $i \in \{1, ..., N\}$ by y_i. Let $y = \sum_{i=1}^{N} y_i$ and $y_{-i} = \sum_{j \neq i} y_j$. Since all blockholders have the same bias, Proposition 1 holds with respect to y. Moreover, given y and $q^*(y)$, trade by dispersed shareholders is also the same as in the baseline model, and in particular, the share price is given by $p^*(y) = \gamma y + v(\mathbb{E}[b], q^*(y))$. The profit of blockholder i is given by

$$\begin{aligned}
\Pi(y_i, y_{-i}) &= (\alpha_i + y_i) v(\beta, q^*(y_{-i}+y_i)) - y_i p^*(y_{-i}+y_i) - \frac{\eta}{2}(y_i)^2 \\
&= \alpha_i v(\beta, q^*(y_{-i}+y_i)) + y_i(\beta - \mathbb{E}[b]) H(q^*(y_{-i}+y_i)) - y_i\gamma y_{-i} - (\gamma+\eta/2)(y_i)^2 \\
&= \alpha_i v_0 + \alpha_i \mathbb{E}[\theta | q > q^*(y_{-i}+y_i)] H(q^*(y_{-i}+y_i)) \\
&\quad ((\alpha_i + y_i)\beta - y_i \mathbb{E}[b]) H(q^*(y_{-i}+y_i)) - (\gamma+\eta/2)(y_i)^2 - y_i\gamma y_{-i}.
\end{aligned}$$

The derivative of $\Pi(y_i, y_{-i})$ with respect to y_i, $\Pi'(y_i, y_{-i})$, is given by

$$\begin{aligned}
\Pi'(y_i, y_{-i}) &= \underbrace{(\beta - \mathbb{E}[b]) H(q^*(y_{-i}+y_i)) - (2\gamma+\eta) y_i - \gamma y_{-i}}_{\text{marginal propensity to buy cash flows}} \\
&\quad \underbrace{+\frac{\partial(-q^*(y_{-i}+y_i))}{\partial y} f(q^*(y_{-i}+y_i)) [\alpha_i (q^*(y_{-i}+y_i)+\beta) + y_i(\beta - \mathbb{E}[b])]}_{\text{marginal propensity to buy votes}}.
\end{aligned}$$

The symmetry across blockholders requires all of them to trade the same amount, and thus, the equilibrium level of y_i^* satisfies

$$\begin{aligned}
0 &= \left[\begin{array}{c} (\beta - \mathbb{E}[b]) H(q^*(Ny_i^*)) - (2\gamma+\eta) y_i^* - \gamma(N-1) y_i^* \\ +\frac{\partial(-q^*(Ny_i^*))}{\partial y} f(q^*(Ny_i^*)) \left[\frac{\alpha}{N}(q^*(Ny_i^*)+\beta) + y_i^*(\beta - \mathbb{E}[b])\right] \end{array} \right] \Leftrightarrow \\
Ny_i^* &= \frac{N}{\gamma(N+1)+\eta}(\beta - \mathbb{E}[b]) H(q^*(Ny_i^*)) + \frac{1}{\gamma(N+1)+\eta} MPV(Ny_i^*).
\end{aligned}$$

The share price in equilibrium is

$$
\begin{aligned}
p^*(N) &= \gamma N y_i^* + v\left(\mathbb{E}[b], q^*(Ny_i^*)\right) \\
&= \gamma\left[\frac{N}{\gamma(N+1)+\eta}\left(\beta-\mathbb{E}[b]\right)H\left(q^*(Ny_i^*)\right)+\frac{1}{\gamma(N+1)+\eta}MPV\left(Ny_i^*\right)\right] \\
&\quad +v\left(\mathbb{E}[b], q^*(Ny_i^*)\right) \\
&= v_0+\left(\frac{\gamma N}{\gamma(N+1)+\eta}\left(\beta-\mathbb{E}[b]\right)+\mathbb{E}[b]\right)H\left(q^*(Ny_i^*)\right) \\
&\quad +\mathbb{E}\left[\theta|q>q^*(Ny_i^*)\right]H\left(q^*(Ny_i^*)\right)+\frac{\gamma}{\gamma(N+1)+\eta}MPV\left(Ny_i^*\right) \\
&= v\left(\frac{\gamma+\eta}{\gamma(N+1)+\eta}\mathbb{E}[b]+\frac{\gamma N}{\gamma(N+1)+\eta}\beta, q^*(Ny_i^*)\right)+\frac{\gamma}{\gamma(N+1)+\eta}MPV\left(Ny_i^*\right) \\
&= v\left(b_{MT}(N), q^*(Ny_i^*)\right)+\frac{\gamma}{\gamma(N+1)+\eta}MPV\left(Ny_i^*\right),
\end{aligned}
$$

as required. ■

Hence, in the limit, the price is the valuation of the blockholders absent any voting premium, that is, $\lim_{N\to\infty} p^*(N) = v\left(\beta, q^*(y_\infty^*)\right)$, where y_∞^* satisfies $y_\infty^* = \frac{1}{\gamma}\left(\beta-\mathbb{E}[b]\right)H\left(q^*(y_\infty^*)\right)$.

C.3.2 Heterogenous blockholders

Suppose μN blockholders have bias $\beta_c \leq -\Delta$, and $(1-\mu)N$ blockholders have bias $\beta_a \geq \Delta$, where $\mu \in (0,1)$ is such that both μN and $(1-\mu)N$ are integers. Thus, blockholders with bias β_c always (i.e., regardless of the realization of signal q) vote against the proposal, and blockholders with bias β_a always vote for the proposal. We let $\overline{\beta} = (1-\mu)\beta_a + \mu\beta_c$ and $q^*(y_c, y_a)$ be the solution of

$$
s_{y_c+y_a,q^*}\left(-q^*\right)+(1-\mu)\alpha+y_a=\tau. \tag{88}
$$

The following result holds.

Proposition 10. *There exist $\overline{\gamma} < \infty$ and $\overline{\eta} < \infty$ such that if $\gamma > \overline{\gamma}$ and $\eta > \overline{\eta}$, then the equilibrium exists and is unique. In equilibrium, blockholders biased in favor (against) the proposal trade y_a^* (y_c^*) shares such that*

$$
\begin{aligned}
N\left(\mu y_c^*+(1-\mu)y_a^*\right) &= \frac{N}{\gamma(N+1)+\eta}\left(\overline{\beta}-\mathbb{E}[b]\right)H\left(q^{**}\right) \\
&\quad +\frac{1}{\gamma(N+1)+\eta}\left(\mu MPV_c^{**}+(1-\mu)MPV_a^{**}\right),
\end{aligned} \tag{89}
$$

and the share price is given by

$$
p^*(N)=v\left(b_{MT}(N,\mu), q^{**}\right)+\frac{\gamma}{\gamma(N+1)+\eta}\left(\mu MPV_c^{**}+(1-\mu)MPV_a^{**}\right), \tag{90}
$$

where

$$\begin{aligned}
q^{**} &= q^*\left(\mu N y_c^*, (1-\mu) N y_a^*\right), \\
b_{MT}(N) &= \frac{\gamma+\eta}{\gamma(N+1)+\eta}\mathbb{E}[b] + \frac{\gamma N}{\gamma(N+1)+\eta}\overline{\beta}, \\
MPV_a^{**} &= \frac{\partial\left(-q^*(y_c, y_a)\right)}{\partial y_a}|_{q^*=q^{**}} f\left(q^{**}\right)\left[\alpha\left(q^{**}+\beta_a\right) + N y_a^*\left(\beta_a - \mathbb{E}[b]\right)\right], \\
MPV_c^{**} &= \frac{\partial\left(-q^*(y_c, y_a)\right)}{\partial y_c}|_{q^*=q^{**}} f\left(q^{**}\right)\left[\alpha\left(q^{**}+\beta_c\right) + N y_c^*\left(\beta_c - \mathbb{E}[b]\right)\right].
\end{aligned}$$

Moreover, as blockholders become more extreme in the sense that β_a increases by ε and β_c decreases by ε, the aggregate MPV, $\mu MPV_c^{**} + (1-\mu) MPV_a^{**}$, increases.

Proof of Proposition 10. We denote the aggregate trade of all blockholders biased against (in favor) of the proposal by y_c (y_a). Given $y = y_c + y_a$, the trade of dispersed investors is as in the baseline model. Given q and the expectations of dispersed shareholders that the proposal will be approved if and only if $q > q_e^*$, the number of affirmative votes by dispersed investors is $s_{y,q_e^*}(q)$. Thus, the proposal is accepted if and only if

$$s_{y,q_e^*}(q) + (1-\mu)\alpha + y_a \geq \tau. \tag{91}$$

Thus, in equilibrium, the median voter is a dispersed investor whose bias $-q^*$ solves

$$\begin{gathered}
s_{y,q^*}(q^*) + (1-\mu)\alpha + y_a = \tau \Leftrightarrow \\
(1-\alpha-y_c-y_a)\left[1 - G(-q^*)\left(1 - \frac{\mathbb{E}[b]-\mathbb{E}[b|b<-q^*]}{\gamma}\frac{H(q^*)}{1-\alpha-y_c-y_a}\right)\right] + (1-\mu)\alpha + y_a = \tau \Leftrightarrow \\
1 - G(-q^*)\left(1 - \frac{\mathbb{E}[b]-\mathbb{E}[b|b<-q^*]}{\gamma}\frac{H(q^*)}{1-\alpha-y_c-y_a}\right) = \frac{\tau-(1-\mu)\alpha-y_a}{1-\alpha-y_c-y_a}.
\end{gathered}$$

If γ is sufficiently high, the solution is unique, as in the baseline model. Suppose this is the case. Then,

$$\frac{\partial(-q^*)}{\partial y_c} = -\frac{G(-q^*)\left(\frac{\mathbb{E}[b]-\mathbb{E}[b|b<-q^*]}{\gamma}\frac{H(q^*)}{(1-\alpha-y_c-y_a)^2}\right) - \frac{\tau-(1-\mu)\alpha-y_a}{(1-\alpha-y_c-y_a)^2}}{\frac{\partial\left(1-G(-q^*)\left(1-\frac{\mathbb{E}[b]-\mathbb{E}[b|b<-q^*]}{\gamma}\frac{H(q^*)}{1-\alpha-y_c-y_a}\right)\right)}{\partial(-q^*)}}.$$

Notice that the denominator is negative (since the solution is unique for a large γ) and that the numerator can be written as

$$\begin{aligned}
& \frac{1}{1-\alpha-y_c-y_a}\left[G(-q^*)\frac{\mathbb{E}[b]-\mathbb{E}[b|b<-q^*]}{\gamma}\frac{H(q^*)}{1-\alpha-y_c-y_a} - \frac{\tau-(1-\mu)\alpha-y_a}{1-\alpha-y_c-y_a}\right] \\
= & \frac{1}{1-\alpha-y_c-y_a}\left[\begin{array}{c} G(-q^*)\frac{\mathbb{E}[b]-\mathbb{E}[b|b<-q^*]}{\gamma}\frac{H(q^*)}{1-\alpha-y_c-y_a} \\ -\left(1-G(-q^*)\left(1-\frac{\mathbb{E}[b]-\mathbb{E}[b|b<-q^*]}{\gamma}\frac{H(q^*)}{1-\alpha-y_c-y_a}\right)\right)\end{array}\right] \\
= & \frac{1}{1-\alpha-y_c-y_a}\left[-1+G(-q^*)\right] < 0,
\end{aligned}$$

so $\frac{\partial(-q^*)}{\partial y_c} < 0$. That is, the median voter becomes more biased against the proposal as blockholders biased against the proposal buy more shares. Next, notice that

$$\begin{aligned}
\frac{\partial\left(-q^*\right)}{\partial y_a} &= -\frac{\frac{1}{1-\alpha-y_c-y_a}\left[G\left(-q^*\right)\frac{\mathbb{E}[b]-\mathbb{E}[b|b<-q^*]}{\gamma}\frac{H(q^*)}{1-\alpha-y_c-y_a}-\frac{\tau-1+\mu\alpha+y_c}{1-\alpha-y_c-y_a}\right]}{\frac{\partial\left(1-G(-q^*)\left(1-\frac{\mathbb{E}[b]-\mathbb{E}[b|b<-q^*]}{\gamma}\frac{H(q^*)}{1-\alpha-y_c-y_a}\right)\right)}{\partial(-q^*)}} \\
&= -\frac{\frac{1}{1-\alpha-y_c-y_a}\left[\frac{\tau-(1-\mu)\alpha-y_a}{1-\alpha-y_c-y_a}-\left(1-G\left(-q^*\right)\right)-\frac{\tau-1+\mu\alpha+y_c}{1-\alpha-y_c-y_a}\right]}{\frac{\partial\left(1-G(-q^*)\left(1-\frac{\mathbb{E}[b]-\mathbb{E}[b|b<-q^*]}{\gamma}\frac{H(q^*)}{1-\alpha-y_c-y_a}\right)\right)}{\partial(-q^*)}} \\
&= -\frac{\frac{1}{1-\alpha-y_c-y_a}G\left(-q^*\right)}{\frac{\partial\left(1-G(-q^*)\left(1-\frac{\mathbb{E}[b]-\mathbb{E}[b|b<-q^*]}{\gamma}\frac{H(q^*)}{1-\alpha-y_c-y_a}\right)\right)}{\partial(-q^*)}} > 0,
\end{aligned}$$

so $\frac{\partial(-q^*)}{\partial y_a} > 0$. That is, the median voter becomes more activist as the activist blockholders buy more shares. Notice that

$$\begin{aligned}
\frac{\partial\left(-q^*\right)}{\partial y_c} &= -\frac{\frac{1}{1-\alpha-y_c-y_a}G\left(-q^*\right)}{\frac{\partial\left(1-G(-q^*)\left(1-\frac{\mathbb{E}[b]-\mathbb{E}[b|b<-q^*]}{\gamma}\frac{H(q^*)}{1-\alpha-y_c-y_a}\right)\right)}{\partial(-q^*)}} \\
&= -\frac{G\left(-q^*\right)}{-1+G\left(-q^*\right)}\frac{\frac{1}{1-\alpha-y_c-y_a}\left[-1+G\left(-q^*\right)\right]}{\frac{\partial\left(1-G(-q^*)\left(1-\frac{\mathbb{E}[b]-\mathbb{E}[b|b<-q^*]}{\gamma}\frac{H(q^*)}{1-\alpha-y_c-y_a}\right)\right)}{\partial(-q^*)}} \\
&= -\frac{G\left(-q^*\right)}{1-G\left(-q^*\right)}\frac{\partial\left(-q^*\right)}{\partial y_a}.
\end{aligned}$$

The profit of an activist blockholder i is given by

$$\begin{aligned}
\Pi_a\left(y_i, y_c, y_{a,-i}\right) &= \left(\alpha_i+y_i\right)v\left(\beta_a, q^*\left(y_c, y_{a,-i}+y_i\right)\right)-y_i p^*\left(y_c+y_{a,-i}+y_i\right)-\frac{\eta}{2}\left(y_i\right)^2 \\
&= \alpha_i v\left(\beta_a, q^*\left(y_c, y_{a,-i}+y_i\right)\right)+y_i\left(\beta_a-\mathbb{E}\left[b\right]\right)H\left(q^*\left(y_c, y_{a,-i}+y_i\right)\right) \\
&\quad -y_i\gamma\left(y_c+y_{a,-i}\right)-\left(\gamma+\eta/2\right)\left(y_i\right)^2 \\
&= \alpha_i v_0+\alpha_i\mathbb{E}\left[\theta|q>q^*\left(y_c, y_{a,-i}+y_i\right)\right]H\left(q^*\left(y_c, y_{a,-i}+y_i\right)\right) \\
&\quad \left(\left(\alpha_i+y_i\right)\beta_a-y_i\mathbb{E}\left[b\right]\right)H\left(q^*\left(y_c, y_{a,-i}+y_i\right)\right)-\left(\gamma+\eta/2\right)\left(y_i\right)^2-y_i\gamma\left(y_c+y_{a,-i}\right),
\end{aligned}$$

and thus, the first order condition implies

$$\left[\begin{array}{c}\left(\beta_a-\mathbb{E}\left[b\right]\right)H\left(q^*\left(y_c, y_{a,-i}+y_i\right)\right)-\left(2\gamma+\eta\right)y_i-\gamma\left(y_c+y_{a,-i}\right)+ \\ \frac{\partial\left(-q^*\left(y_c, y_{a,-i}+y_i\right)\right)}{\partial y_a}f\left(q^*\left(y_c, y_{a,-i}+y_i\right)\right)\left[\alpha_i\left(q^*\left(y_c, y_{a,-i}+y_i\right)+\beta_a\right)+y_i\left(\beta_a-\mathbb{E}\left[b\right]\right)\right]\end{array}\right]=0.$$

Similarly, the profit of a blockholder i biased against the proposal is given by

$$\begin{aligned}
\Pi_c\left(y_i, y_{c,-i}, y_a\right) &= \alpha_i v_0+\alpha_i\mathbb{E}\left[\theta|q>q^*\left(y_{c,-i}+y_i, y_{a,}\right)\right]H\left(q^*\left(y_{c,-i}+y_i, y_{a,}\right)\right) \\
&\quad \left(\left(\alpha_i+y_i\right)\beta_c-y_i\mathbb{E}\left[b\right]\right)H\left(q^*\left(y_{c,-i}+y_i, y_{a,}\right)\right)-\left(\gamma+\eta/2\right)\left(y_i\right)^2-y_i\gamma\left(y_{c,-i}+y_a\right)
\end{aligned}$$

and thus, the first order condition implies

$$\left[\begin{array}{c}\left(\beta_c-\mathbb{E}\left[b\right]\right)H\left(q^*\left(y_{c,-i}+y_i,y_{a,}\right)\right)-\left(2\gamma+\eta\right)y_i-\gamma\left(y_{c,-i}+y_a\right)+\\ \frac{\partial\left(-q^*\left(y_{c,-i}+y_i,y_{a,}\right)\right)}{\partial y_c}f\left(q^*\left(y_{c,-i}+y_i,y_{a,}\right)\right)\left[\alpha_i\left(q^*\left(y_{c,-i}+y_i,y_{a,}\right)+\beta_c\right)+y_i\left(\beta_c-\mathbb{E}\left[b\right]\right)\right]\end{array}\right]=0.$$

The symmetry across blockholders with bias β_a implies that they all choose y_a^*, and the symmetry across blockholders with bias β_c implies that they all choose y_c^*. Let

$$q^{**}\equiv q^*\left(\mu N y_c^*,\left(1-\mu\right)N y_a^*\right).$$

Then, the two FOC conditions are reduced to

$$\begin{array}{c}\left(\beta_a-\mathbb{E}\left[b\right]\right)H\left(q^{**}\right)-\left(\gamma+\eta\right)y_a^*-\gamma\left(\mu y_c^*+\left(1-\mu\right)y_a^*\right)N\\ +\frac{\partial\left(-q^{**}\right)}{\partial y_a}f\left(q^{**}\right)\left[\frac{\alpha}{N}\left(q^{**}+\beta_a\right)+y_a^*\left(\beta_a-\mathbb{E}\left[b\right]\right)\right]=0\end{array}$$

and

$$\begin{array}{c}\left(\beta_c-\mathbb{E}\left[b\right]\right)H\left(q^{**}\right)-\left(\gamma+\eta\right)y_c^*-\gamma\left(\mu y_c^*+\left(1-\mu\right)y_a^*\right)N\\ +\frac{\partial\left(-q^{**}\right)}{\partial y_c}f\left(q^{**}\right)\left[\frac{\alpha}{N}\left(q^{**}+\beta_c\right)+y_c^*\left(\beta_c-\mathbb{E}\left[b\right]\right)\right]=0.\end{array}$$

The price is

$$p^*=\gamma\left(\mu N y_c^*,\left(1-\mu\right)N y_a^*\right)+v\left(\mathbb{E}\left[b\right],q^*\left(\mu N y_c^*,\left(1-\mu\right)N y_a^*\right)\right).$$

Multiplying the FOC of blockholders with bias β_a by $(1-\mu)$ and the FOC of blockholders with bias β_c by μ, and adding the two outcomes, we get

$$\begin{aligned}&\left[\mu y_c^*+\left(1-\mu\right)y_a^*\right]N\\ =\;&\frac{N}{\gamma\left(N+1\right)+\eta}\left(\left(1-\mu\right)\beta_a+\mu\beta_c-\mathbb{E}\left[b\right]\right)H\left(q^{**}\right)\\ &+\frac{1}{\gamma\left(N+1\right)+\eta}\left[\begin{array}{c}\left(1-\mu\right)\frac{\partial\left(-q^{**}\right)}{\partial y_a}f\left(q^{**}\right)\left[\alpha\left(q^{**}+\beta_a\right)+Ny_a^*\left(\beta_a-\mathbb{E}\left[b\right]\right)\right]\\ +\mu\frac{\partial\left(-q^{**}\right)}{\partial y_c}f\left(q^{**}\right)\left[\alpha\left(q^{**}+\beta_c\right)+Ny_c^*\left(\beta_c-\mathbb{E}\left[b\right]\right)\right]\end{array}\right]\\ =\;&\frac{N}{\gamma\left(N+1\right)+\eta}\left(\overline{\beta}-\mathbb{E}\left[b\right]\right)H\left(q^{**}\right)+\frac{1}{\gamma\left(N+1\right)+\eta}\left[\mu MPV_c^{**}+\left(1-\mu\right)MPV_a^{**}\right].\end{aligned}$$

Thus, we can write the share price as

$$p^*=v\left(b_{MT}\left(N\right),q^{**}\right)+\frac{1}{\gamma\left(N+1\right)+\eta}\left[\mu MPV_c^{**}+\left(1-\mu\right)MPV_a^{**}\right].$$

Next, notice that

$$\begin{aligned}\lim_{\gamma\to\infty}\frac{\partial\left(-q^*\right)}{\partial y_a}&=\frac{1}{1-\alpha}\frac{G\left(-q^*\right)}{g\left(-q^*\right)}>0,\\ \lim_{\gamma\to\infty}\frac{\partial\left(-q^*\right)}{\partial y_c}&=-\frac{G\left(-q^*\right)}{1-G\left(-q^*\right)}\frac{1}{1-\alpha}\frac{G\left(-q^*\right)}{g\left(-q^*\right)}<0,\end{aligned}$$

and thus,

$$\lim_{\gamma\to\infty} MPV_a^{**} = \frac{1}{1-\alpha}\frac{G(-q^{**})}{g(-q^{**})} f(q^{**}) \frac{\alpha}{N}(q^{**}+\beta_a) > 0,$$
$$\lim_{\gamma\to\infty} MPV_c^{**} = -\frac{G(-q^{**})}{1-G(-q^{**})}\frac{1}{1-\alpha}\frac{G(-q^{**})}{g(-q^{**})} f(q^{**}) \frac{\alpha}{N}(q^{**}+\beta_c) > 0.$$

Also notice that $\lim_{\gamma\to\infty} q^{**}$ solves

$$1-G(-q^{**}) = \frac{\tau-(1-\mu)\alpha}{1-\alpha}.$$

Thus,

$$\begin{aligned}\lim_{\gamma\to\infty}\left[\mu MPV_c^{**}+(1-\mu)MPV_a^{**}\right] &= \frac{1}{1-\alpha}\frac{G(-q^{**})}{g(-q^{**})} f(q^{**}) \frac{\alpha}{N} \\ &\times\left[q^{**}+(1-\mu)\beta_a+\mu\beta_c-\mu\frac{q^{**}+\beta_c}{1-G(-q^{**})}\right].\end{aligned}$$

Suppose β_a increases by $\varepsilon > 0$ and β_c decreases by ε. Then, we can write

$$\begin{aligned}\lim_{\gamma\to\infty}\left[\mu MPV_c^{**}+(1-\mu)MPV_a^{**}\right] &= \frac{1}{1-\alpha}\frac{G(-q^{**})}{g(-q^{**})} f(q^{**}) \frac{\alpha}{N} \\ &\times\left[\begin{array}{c} q^{**}+(1-\mu)\beta_a+\mu\beta_c-\mu\frac{q^{**}+\beta_c}{1-G(-q^{**})} \\ +(1-\mu)\varepsilon-\mu\varepsilon+\mu\frac{\varepsilon}{1-G(-q^{**})}\end{array}\right].\end{aligned}$$

The derivative of $\lim_{\gamma\to\infty}\left[\mu MPV_c^{**}+(1-\mu)MPV_a^{**}\right]$ with respect to ε is proportional to

$$1-2\mu+\frac{\mu}{1-G(-q^{**})} = 1-2\mu+\frac{\mu(1-\alpha)}{\tau-\alpha+\alpha\mu}.$$

Notice that

$$1-2\mu+\frac{\mu(1-\alpha)}{\tau-\alpha+\alpha\mu} > 0 \Leftrightarrow \tau-\alpha+(1-2(\tau-\alpha))\mu-2\alpha\mu^2 > 0.$$

This concave expression is positive both when $\mu = 0$ and when $\mu = 1$ (since $\alpha < 1-\tau$), and thus it is positive for any $\mu \in [0,1]$. Therefore, $\lim_{\gamma\to\infty}\left[\mu MPV_c^{**}+(1-\mu)MPV_a^{**}\right]$ increases in ε, as required. ■

C.4 Block trading

The block trading premium has been analyzed by financial economists at least since Barclay and Holderness (1989) and has been used to measure the private benefits of control enjoyed by large shareholders. For example, Dyck and Zingales (2004) regard it as an alternative and sometimes advantageous empirical strategy to the dual-class share premium for measuring private benefits.

We therefore augment our baseline model with a pre-stage of block trading in the spirit of Burkart, Gromb, and Panunzi (2000) or Albuquerque and Schroth (2010). Specifically, at the first stage, an incumbent blockholder (I) with bias β_I and Nash bargaining power δ negotiates a trade of the block α with a rival blockholder (R) with bias β_R and Nash bargaining power $1-\delta$. After that, the investor who emerges as the owner of the block, trades with dispersed shareholders and then votes his shares as in the baseline model. The other investor exits the game.

We let $\Pi^*(\beta) \equiv \Pi^*(y^*(\beta);\beta)$ be the blockholder's expected profit, given the optimal trade $y^*(\beta)$ and bias β. If $\Pi^*(\beta_I) \geq \Pi^*(\beta_R)$, then I and R do not trade with each other, so I owns the block and realizes payoff $\Pi^*(\beta_I)$. Otherwise, I and R trade, and R pays a price per share of

$$p_B = \frac{1}{\alpha}\left(\Pi^*(\beta_I) + \delta\left(\Pi^*(\beta_R) - \Pi^*(\beta_I)\right)\right) \tag{92}$$

for the block. In Section C.4.1 below, we provide a formal analysis of this extended model. There, we first establish that for sufficiently large γ and η, the profit $\Pi^*(\beta)$ is strictly increasing in β. The reason is that more activist blockholders value the proposal more. Hence, the blockholder with the larger, more activist bias ends up owning the controlling block. If we allowed for multiple potential bidders for the controlling block, the model would predict that the most activist bidder would end up owning the block.

Second, we show that the block premium, defined as the difference between the block trade price and the subsequent market price, is always negative if $\beta < \mathbb{E}[b]$. However, it is always positive if $\beta > \mathbb{E}[b]$, the block is sufficiently small, and the market for block trades is sufficiently competitive, which in our setup means that the incumbent has sufficient bargaining power, i.e., δ is large enough. Hence, the model can accommodate positive as well as negative block premiums, which is consistent with the empirical evidence.[36]

The discussion above also applies to private placements of blocks, in which a firm issues equity and places the new shares as a block with a single investor. Wruck (1989), Hertzel and Smith (1993), and Barclay, Holderness, and Sheehan (2007) study samples of such private placements. Our analysis implies that the firm would auction of the block to the most activist blockholder, and that the blockholder might pay a premium or accept a discount relative to the price paid by dispersed shareholders.

C.4.1 Analysis of block trading

By definition, $\Pi^*(\beta) = \Pi(y^*(\beta), \beta)$, where $\Pi(y, \beta)$ is given by (17), $y^*(\beta)$ by (21), and $p^*(y,\beta) = \gamma y + v(\mathbb{E}[b], q^*(y,\beta))$. To ease the exposition we let $q^*(\beta) = q^*(y^*(\beta), \beta)$ and $p^*(\beta) = p^*(y^*(\beta), \beta)$. Thus,

$$\begin{aligned}
\Pi^*(\beta) &= (\alpha + y^*(\beta))\, v(\beta, q^*(\beta)) - y^*(\beta)\, p^*(\beta) - \frac{\eta}{2}(y^*(\beta))^2 \\
&= \alpha v(\beta, q^*(\beta)) + y^*(\beta)(\beta - E[b])\, H(q^*(\beta)) - (\gamma + \eta/2)(y^*(\beta))^2 \\
&= \alpha v(\beta, q^*(\beta)) + \frac{1}{2}\frac{1}{2\gamma+\eta}\left[(\beta - \mathbb{E}[b])^2 H(q^*(\beta))^2 - MPV\,(y^*(\beta))^2\right]
\end{aligned}$$

[36]See Albuquerque and Schroth (2010), Table 2, which reports a negative median block premium, and Albuquerque and Schroth (2015), Figure 1, which shows that 53 of 114 block trades in their sample have a negative premium.

The first term in the blockholder's equilibrium payoff is the value of his endowment given $q^*(\beta)$, whereas the second term represents the gains from trade.

Proposition 11. *There exist $\overline{\gamma} < \infty$ and $\overline{\eta} < \infty$ such that if $\gamma > \overline{\gamma}$ and $\eta > \overline{\eta}$, then the blockholder's equilibrium payoff strictly increases in β.*

Proof of Proposition 11. In the proof of Proposition 2, we show that if $\beta \neq G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})$ ($\beta \neq G^{-1}(\frac{1-\tau}{1-\alpha})$) and γ is large then $\beta \neq \beta_L(y^*(\beta))$ ($\beta \neq \beta_H(y^*(\beta))$). Then,

$$\begin{aligned}
\frac{\partial \Pi^*(\beta)}{\partial \beta} &= \frac{\partial \Pi(y^*(\beta),\beta)}{\partial y}\Big|_{y=y^*(\beta)} \cdot \frac{\partial y^*(\beta)}{\partial \beta} + \frac{\partial \Pi(y^*(\beta),\beta)}{\partial \beta} \\
&= 0 \cdot \frac{\partial y^*(\beta)}{\partial \beta} + \frac{\partial \Pi(y^*(\beta),\beta)}{\partial \beta} \\
&= (\alpha + y^*(\beta)) \frac{\partial v(\beta, q^*(y^*(\beta),\beta))}{\partial \beta} - y^*(\beta) \frac{\partial p^*(y^*(\beta),\beta)}{\partial \beta} \\
&= (\alpha + y^*(\beta)) \left[\frac{\partial v(\beta, q^*(y^*(\beta),\beta))}{\partial \beta} + \frac{\partial v(\beta, q^*(y^*(\beta),\beta))}{\partial q^*} \frac{\partial q^*(y^*(\beta),\beta)}{\partial \beta} \right] \\
&\quad - y^*(\beta) \frac{\partial v(\mathbb{E}[b], q^*(y^*(\beta),\beta))}{\partial q^*} \frac{\partial q^*(y^*(\beta),\beta)}{\partial \beta}
\end{aligned}$$

If $G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}) < \beta < G^{-1}(\frac{1-\tau}{1-\alpha})$ then for large γ we have $q^*(y,\beta) = -\beta$. Thus,

$$\begin{aligned}
\frac{\partial v(\beta,-\beta)}{\partial \beta} &= \frac{\partial}{\partial \beta} \int_{-\beta} (\theta + \beta) f(q)\, dq \\
&= H(-\beta) > 0
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial p^*(y^*(\beta),\beta)}{\partial \beta} &= \frac{\partial v(\mathbb{E}[b],-\beta)}{\partial \beta} \\
&= \frac{\partial}{\partial \beta} \int_{-\beta} (\theta + \mathbb{E}[b]) f(q)\, dq \\
&= -(\beta - \mathbb{E}[b]) f(-\beta)
\end{aligned}$$

Thus,

$$\frac{\partial \Pi^*(\beta)}{\partial \beta} = (\alpha + y^*(\beta)) H(-\beta) + y^*(\beta)(\beta - \mathbb{E}[b]) f(-\beta)$$

Recall that $\lim_{\gamma \to \infty} y^*(\beta) = 0$, and thus, for large γ we have $\frac{\partial \Pi^*(\beta)}{\partial \beta} \approx \alpha H(-\beta) > 0$, that is, the first order effect is on the blockholder endowment rather than on his gains from trade.

If $\beta < G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})$ $(G^{-1}(\frac{1-\tau}{1-\alpha}) < \beta)$ then $q^*(y^*(\beta),\beta) = \beta_L(y^*(\beta))$ $(q^*(y^*(\beta),\beta) = \beta_H(y^*(\beta)))$ does not depend on β directly, that is, $\frac{\partial q^*(y^*(\beta),\beta)}{\partial \beta} = 0$. And in this case,

$$\begin{aligned}\frac{\partial \Pi^*(\beta)}{\partial \beta} &= (\alpha + y^*(\beta)) \frac{\partial v(\beta, q^*(y^*(\beta),\beta))}{\partial \beta} \\ &= (\alpha + y^*(\beta)) H(q^*(y^*(\beta),\beta)) \\ &> 0\end{aligned}$$

Since $\Pi^*(\beta)$ is continuous in β when $\beta \in \left\{G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}), G^{-1}(\frac{1-\tau}{1-\alpha})\right\}$, $\Pi^*(\beta)$ increases globally in β. ■

Suppose a block of α shares is acquired by a bidder with a bias β who paid $\Pi^*(\beta) - \Delta\alpha$ where $\Delta > 0$. Parameter Δ captures the competitiveness of the block market, where smaller Δ implies more competitiveness. Recall the share price is given by $p^*(y^*(\beta)) = \gamma y^*(\beta) + v(\mathbb{E}[b], q^*(y^*(\beta),\beta))$, then the block premium is

$$P_B \equiv \Pi^*(\beta)/\alpha - \Delta - p^*(y^*(\beta),\beta) \tag{93}$$

Here, we replace $\Pi^*(\beta_R)$ in the text with $\Pi^*(\beta)$ and the discount blockholder R receives on the block price, $(1-\delta)(\Pi^*(\beta_R) - \Pi^*(\beta_I))$ with the constant Δ.

Proposition 12. *There exist $\overline{\gamma} < \infty$ and $\overline{\eta} < \infty$ such that if $\gamma > \overline{\gamma}$ and $\eta > \overline{\eta}$, then*

(i) If $\beta \leq \mathbb{E}[b]$ then the equilibrium block premium is strictly negative.

(ii) If $\beta > \mathbb{E}[b]$ then there exist $\alpha^* > 0$ and $\Delta^* > 0$ such that if $\alpha \in (0,\alpha^*)$ and $\Delta \in (0,\Delta^*)$ then the equilibrium block premium is strictly positive.

Proof of Proposition 12. Notice that

$$\begin{aligned}\lim_{\gamma\to\infty} P_B &= \lim_{\gamma\to\infty} \Pi^*(\beta)/\alpha - \Delta - \lim_{\gamma\to\infty} p^*(y^*(\beta),\beta) \\ &= v(\beta, q^*(0,\beta)) - v(\mathbb{E}[b], q^*(0,\beta)) - \lim_{\gamma\to\infty}[\gamma y^*(\beta)] - \Delta \\ &= (\beta - \mathbb{E}[b]) H(q^*(0,\beta)) - \frac{1}{2}\left[\lim_{\gamma\to\infty} MPV(y^*) + (\beta - \mathbb{E}[b]) H(q^*(0,\beta))\right] - \Delta \\ &= \frac{1}{2}\left[(\beta - \mathbb{E}[b]) H(q^*(0,\beta)) - \lim_{\gamma\to\infty} MPV(y^*)\right] - \Delta\end{aligned}$$

Recall $\lim_{\gamma\to\infty} MPV(y^*) \geq 0$. Thus, if $\beta \leq \mathbb{E}[b]$ then $\lim_{\gamma\to\infty} P_B < 0$. Suppose $\beta > E[b]$. Thus, $\frac{1}{2}(\beta - \mathbb{E}[b]) H(q^*(0,\beta)) - \Delta$ is bounded away from zero, and if $\lim_{\gamma\to\infty} MPV(y^*)$ is sufficiently small then $\lim_{\gamma\to\infty} P_B > 0$. There are three cases to consider:

1. If in addition $G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}) < \beta < G^{-1}(\frac{1-\tau}{1-\alpha})$ then $\lim_{\gamma\to\infty} MPV(y^*) = 0$.

2. If in addition $\beta < G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})$ then $q^*(0,\beta) = -G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})$, and

$$\lim_{\gamma\to\infty} MPV(y^*) = \frac{\tau}{1-\alpha} \frac{f(-G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}))}{g(G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}))} \frac{\alpha}{1-\alpha}(G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}) - \beta)$$

Notice that $\lim_{\alpha \to 0} [\lim_{\gamma \to \infty} MPV(y^*)] = 0$, and hence, there exists α^* as required.

3. Suppose $\beta > G^{-1}(\frac{1-\tau}{1-\alpha})$. Then, $q^*(0, \beta) = -G^{-1}(\frac{1-\tau}{1-\alpha})$, and

$$\lim_{\gamma \to \infty} MPV(y^*) = \frac{1-\tau}{1-\alpha} \frac{f(-G^{-1}(\frac{1-\tau}{1-\alpha}))}{g(G^{-1}(\frac{1-\tau}{1-\alpha}))} \frac{\alpha}{1-\alpha} (\beta - G^{-1}(\frac{1-\tau}{1-\alpha})).$$

Notice that $\lim_{\alpha \to 0} [\lim_{\gamma \to \infty} MPV(y^*)] = 0$, and hence, there exists α^* as required.

■